Drug approval decision times, international reference pricing and access to new drugs

Nicolas HOUY[†] Izabela JELOVAC[‡]

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Abstract

This paper analyzes how drug approval procedures influence the incentives of pharmaceutical firms to launch new drugs in the presence of international reference pricing. First, we show that the number of countries in which a firm commercializes a new drug is larger when countries do not approve this new drug simultaneously. Furthermore, we show that a firm's incentives to launch a new drug in one or another country are the same if the drug approval times are identical across countries or if the difference between approval times are small enough. However, we show that these incentives can change if the approval times differences across countries are larger. Last, we show in a simplified framework that countries should be in favor of a non-centralized drug approval procedure.

Key-Words: Drug approval, access to new drugs, international reference pricing.

JEL Classification: I11, L65.

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[†]University of Lyon, Lyon, F-69007, France; CNRS, GATE Lyon Saint-Etienne, F-69130, France. E-mail: houy@gate.cnrs.fr.

[‡]University of Lyon, Lyon, F-69007, France; CNRS, GATE Lyon Saint-Etienne, F-69130, France. E-mail: jelovac@gate.cnrs.fr.

1 Introduction

This paper analyzes how drug approval procedures influence the incentives of pharmaceutical firms to launch new drugs in different countries under international reference pricing. The general focus is on the exogenous timing of drug approvals, the endogenous choice of countries for the launches of new drugs and on the regulated pricing of drugs. We also analyze in a simple framework the preferences of countries for approval procedures. We leave apart the important issues of quality and safety on the one hand and of incentives to innovate on the other hand, both sensitive to the drug approval regulation.

Before launching a new drug on a market, a pharmaceutical firm must satisfy some regulatory constraints. One important compulsory step is to obtain a drug approval (or marketing authorization). This drug approval depends on the proof that the new drug complies with safety, quality and efficacy standards. In the US, the Food and Drug Administration is in charge of approving new drugs. In the EU, four alternative procedures co-exist for drug approval: the centralized procedure, the decentralized procedure, the national procedure and the mutual recognition procedure [Eudralex, 2013]. The centralized procedure allows firms to submit a single application to the European Medicines Agency to obtain a centralized drug approval valid in all EU countries, Iceland, Liechtenstein and Norway.[1] The decentralized procedure may be used to obtain a drug approval in several Member States when the applicant does not yet have a drug approval in any country. The national procedure is used to obtain a drug approval in one country at a time or in the initial phase of the mutual recognition procedure. The mutual recognition procedure is used to request a drug approval in EU countries for products that have already received approvals in other EU countries.

[1] The centralized procedure is compulsory for all medicines derived from biotechnology and other high-tech processes, as well as for human medicines for the treatment of HIV/AIDS, cancer, diabetes, neurodegenerative diseases, auto-immune and other immune dysfunctions, and viral diseases, and for veterinary medicines for use for growth or yield enhancers. The centralized procedure is also open to products that bring a significant therapeutic, scientific or technical innovation, or is in any other respect in the interest of patient or animal health. As a result, the majority of genuinely novel medicines are authorized through the European Medicines Agency.

The distinctive feature between these procedures that motivates our analysis is whether the drug is approved simultaneously in all countries or not. In the case of the EU, only the centralized procedure guarantees a simultaneous drug approval in all EU countries. In the alternative three non-centralized procedures, new drugs do not necessarily obtain an approval simultaneously in all countries. Once a drug approval is obtained, a drug producer can start commercializing its new drugs at any time, but without exceeding three years (sunset clause). We show that whether drug approvals are simultaneous or not can influence how pharmaceutical firms strategically respond to international reference pricing. In particular, it influences whether they launch a new drug in different countries where they have a drug approval.

A second set of regulatory constraints that is crucial when launching a new drug and after, refers to pricing rules. In fact, the system of patents together with the widespread availability of health insurance (public or private) can induce excessively high pharmaceutical prices in the absence of regulation. Therefore, many countries use some regulatory tools to control the expenses related to the consumption of drugs.

One such regulation is the international reference pricing (IRP). IRP in a country imposes a price cap based on prices of identical drugs in other reference countries. Almost every EU country uses IRP. The IRP formula varies from one country to another. Some countries use the lowest price observed in the reference countries while other use an average of the reference prices. Countries can revise prices periodically and they choose to use foreign prices retroactively or not, [Rankin, 2003]. The basket of reference countries varies in size from one country to another. We observe that the sets of reference countries include an increasing number of countries over time, [Leopold *et al.*, 2012]. This regulation typically induces a convergence between international drug prices. Moreover, many authors argue that it gives incentives to pharmaceutical firms to sequentially launch new drugs, initiating the sales in high-price countries, [Varol *et al.*, 2012, among others].

The literature on IRP is, up to our knowledge, mainly empirical. It repeatedly provides evidence about the link between IRP, interdependence of prices and timing of new drugs launches. In particular, several authors show that price regulations such as IRP cause launch delays and even absence of launches in some countries, [Danzon and Epstein, 2008; Danzon *et al.*, 2005; Kyle, 2007; Lanjouw, 2005; Rankin, 2003; Varol *et al.*, 2012]. However, they do not explain why some launches are delayed in time while other launches simply never occur in some countries. [Verniers *et al.*, 2011] analyze launch delays for drugs, considering that launch delays are a regulator's strategic decision rather than a firm's one.

[Houy and Jelovac, 2015] is the only article to theoretically derive the optimal firms' strategy of launch timing. It focuses on the network that the referencing of prices between countries generates and on varying modalities of IRPs. It shows that, for a rather flexible structure of such network (a transitive one), firms respond to IRP by never launching drugs sequentially if IRP policies are retroactive and prices are revised periodically. Instead, under these conditions, firms are better off launching the new drug immediately in countries with a high willingness to pay and never in the remaining ones. Counter-examples are also provided for some specific network structures where IRP highly connects one country to many others. An implicit assumption in [Houy and Jelovac, 2015] is the absence, or equivalently, simultaneity, of drug approval times in all countries.

In the present paper, we switch focus to consider access to new drugs. Concretely, we show that the drug approval procedure can be determinant for the number of countries where a firm strategically launches new drugs under IRP. To do so, we conveniently use a simple dynamic n-countries model as in [Houy and Jelovac, 2015], relaxing the assumption of simultaneous drug approval times in all countries. We consider retroactive IRP with periodical price revisions. To focus on the very effects of approval times, we abstract from any network effect of IRP by assuming that it is complete: Every country references the prices of every other country. In such a setting, a two-country case allows presenting and explaining our main results in a simple way. We extend our results to a n-country setting in separate sections, which are a fortiori more formal and technical.

The firm faces the following trade off when deciding whether and when to launch the new drug in a given country: The gains (present and future) from selling in this country against the future losses from spreading through IRP the price of this country to other countries that would have paid higher prices otherwise. As a result, firms have incentives to forego sales in low-price markets, [Houy and Jelovac, 2015].

However, we show in the present paper that these incentives are very sensitive to the timing of drug approvals. For example, some countries may not be served if their approval time is close to the one of higher-price countries. However, they may be served if approval times are further apart. The reason is that the gains from selling in an additional market concern a higher number of periods while the losses from spreading a low price through IRPs does not vary. Formally, we show that the number of countries in which a firm commercializes a new drug is larger when countries do not approve this new drug simultaneously. Furthermore, we show that a firm's incentives to launch a new drug in a country are the same no matter whether the MA times are simultaneous across countries or if they are close enough. However, we show that these incentives can change if the differences between approval times increase.

In addition, restricting the number of countries to two for tractability, we provide the following argument about the resulting countries' preferences between a centralized and a non-centralized approval procedure. A high-price country is, in theory, better-off approving the new drug sufficiently later than the low-price country to have a chance benefiting from a low reference price. The low-price country is indifferent because its price never gets lower than its willingness to pay. Therefore, our conjecture is that countries are in favor of a non-centralized approval procedure. The firm in theory prefers the drug to be approved as soon as possible.

Our analysis is a variation on the well-known issues related to dynamic monopoly pricing and best-price guarantees. Dynamic monopoly pricing is generally analyzed in the context of a market for durable goods. Depending on the degree of patience of the seller and the buyers, dynamic monopoly pricing can result in marginal-cost pricing because of a time inconsistency problem (the Coase conjecture, [Coase, 1972]) or in perfect price discrimination (the Pacman conjecture, [Bagnoli et al., 1989]). The best-price guarantee is generally analyzed in a context of competition between sellers and it is shown to favor tacit collusion and high prices, [Sargent, 1993]. When analyzed in the context of a monopolistic seller, the best-price guarantee can be shown to alleviate the time inconsistency problem of a dynamic monopoly selling a durable good, [Cooper and Fries, 1991]. However, the existing results from this rich literature cannot be simply transposed to our question about dynamic drug launching and pricing under IRP, mainly because of two reasons: First, we do not consider a durable good and thus past profits do not "eat" future profits. Second, we consider that the buyers (the countries) impose an IRP policy while the existing literature considers that the monopoly seller decides to use a best-price guarantee or not.

The remaining of the paper is organized as follows. In Section 2, we describe the formal framework. In Section 3, we use a simple two-country setting to present and explain our main results. In Section 4, we derive general results on optimal pricing. In Section 5, we formally extend our results about approval times effects to a n-country setting. Section 6 concludes. Proofs are given in the Appendix.

2 Formal framework

We consider the optimal price vectors for a monopolistic firm offering a new drug for the international market. The patent that protects any new drug justifies the monopolistic position of the firm. With no lack of generality, we consider that the cost of production for the drug is null. Buyers are countries or the health authorities in each of the countries. Let $N = (1, ..., N)$ be the set of countries.[2] We assume that the drug is sold in all countries with perfect segmentation. Said differently, there are no parallel imports.

Each country i has a willingness to pay (WTP), w_i, that is the price above which it

[2]With a slight lack a rigor, but with no risk of confusion, N is both the set of countries and its cardinality.

is not willing to buy the drug under any circumstances. Let $(w_1, ..., w_N) \in {\mathbb{R}^{+*}}^N$ be the WTPs for all countries.[3]

Each country i is also characterized by a market size (MS), ω_i. This is the quantity the seller can sell in country i if country i buys the drug. Let $(\omega_1, ..., \omega_N) \in {\mathbb{R}^{+*}}^N$ be the MSs for all countries.[4]

Finally, each country i allows the monopoly to enter its market only after $m_i \in \mathbb{N}$ periods corresponding to the time needed for all processes of authorization, application, referring, filing, approval, etc in country i. Let $(m_1, ..., m_N) \in \mathbb{N}$ be the Approval Times (ATs) for all countries.

The problem of the monopolistic seller is to maximize its intertemporal profit over the price vectors $(p_i^t)_{i\in N, t\in\mathbb{N}}$ where $\forall i \in N, \forall t \in \mathbb{N}, p_i^t \in \mathbb{R}^+$ is the price set in country i at time t. Let $\mathcal{P}$ be the set of all possible price vectors. Notice that, obviously, we allow for prices that would change over time and countries.

We consider that all countries are part of the same complete international reference pricing system, hence all IRPs are complete.[5] IRPs are retroactive, they are based on the lowest price abroad and prices can be revised over time. Then, given a price vector $(p_i^t)_{i\in N, t\in\mathbb{N}} \in \mathcal{P}$, for any period t, the demand in country i is ω_i if the following conditions are all met:

1. $t \geq m_i$. If this condition is not met, the drug is not approved in country i and cannot enter the market.

2. $p_i^t \leq w_i$. If this condition is not met, the price of the drug in country i is higher than the WTP of this country.

3. if $t > 0$, $p_i^t \leq \min_{i'\in N, t'<t} p_{i'}^{t'}$. This condition corresponds to the constraint imposed

[3]As a general remark, we use superscripts for time indices and subscripts for country indices. $\mathbb{R}^+$ denotes the set of all positive real numbers. $\mathbb{R}^{+*}$ denotes the set of all strictly positive real numbers. $\mathbb{N}$ denotes the set of all positive integers.

[4]Notice that, with no loss of generality, we do not consider countries with null WTP or MS. Such countries can just be considered nonexistent by the seller.

[5]For a thorough study of incomplete IRPs, see [Houy and Jelovac, 2015].

by the IRPs. We do not explicitly model the bargaining process between the seller and the buyer but consider, in line with what is observed in reality, that country i can impose that the price at which the drug is sold in its territory be not higher than any price ever observed in any country since its IRP is complete (every country references each others' prices when available).

If any of the previous conditions is not met, the quantity sold at time t, in country i is null.[6] For any price vector $P = (p_i^t)_{i \in N, t \in \mathbb{N}} \in \mathcal{P}$, we define the set of countries where the drug is sold at time t as $S^t(P)$. Formally,

Definition 1

$\forall P = (p_i^t)_{i \in N, t \in \mathbb{N}} \in \mathcal{P}, S^t(P) = \{i \in N, t \geq m_i \; and \; p_i^t \leq w_i \; and \; [t > 0 \Rightarrow p_i^t \leq \min_{i' \in N, t' < t} p_{i'}^{t'}]\}$.

Then, the intertemporal profit earned by the monopolistic seller is given by

$$\Pi(P) = \sum_{t \in \mathbb{N}} \beta^t \sum_{i \in S^t(P)} p_i^t \omega_i,$$

where $0 < \beta < 1$ is the time discount rate. Let $P^a \subseteq \mathcal{P}$ be the set of price vectors maximizing the seller's profit,

$$P^a = \arg\max_{P \in \mathcal{P}} \Pi(P).$$

3 Simple two-country case

In this simple case, two countries have WTPs (w_1, w_2) for the new drug, with $w_1 < w_2$, MSs (ω_1, ω_2) and ATs (m_1, m_2). They apply IRP in a complete manner: They both reference each other's price, if the latter is available.

Let us consider the monopolistic producer of a new drug. We assume that the producer has no influence on the approval procedure, that is, on the ATs. However, the producer can choose whether and when to commercialize the drug in a country, as soon as the AT is

[6] We will say that the drug is sold in a country when the quantity sold is strictly positive.

reached. The firm's monopolistic position allows the setting of prices so as to fully extract the countries surplus. In every period of time, the firm decides whether to launch its new drug in a given country weighting the gains (present and future) from selling in this country against the future losses from spreading through IRP the price of this country to other countries that would have paid higher prices otherwise.

The results obtained in [Houy and Jelovac, 2015] for complete IRP networks can easily be adapted to this simple two-country setting: (1) There is no withdrawal of the new drug in any country where the drug has already been sold because the firm would not forego profits from a market that has already spread its price to others; (2) Whenever a drug is sold in a country, it is also sold in all countries with larger WTPs because sales in the latter do not spread low prices and they bring profits to the firm; (3) There is no strict incentive to delay the drug launch in any country after the AT because the same trade off between new gains and losses from price spreading is repeated in every period of time. Concretely, the firm either sells in a country from its AT onwards, or it never sells there.

These existing results allow us to write the firm's profit as follows.

If the firm sells the drug in both countries from their respective ATs onwards, its intertemporal profits are π_{12}, with

$$\pi_{12} = \begin{cases} \frac{\beta^{m_1}}{1-\beta} w_1\omega_1 + \frac{\beta^{m_2}}{1-\beta} w_1\omega_2 & \text{, if } m_1+1 \leq m_2 \\ \frac{\beta^{m_2}-\beta^{m_1+1}}{1-\beta} w_2\omega_2 + \frac{\beta^{m_1}}{1-\beta} w_1(\omega_1+\beta\omega_2) & \text{, if } m_1+1 \geq m_2 \end{cases}.$$

IRP makes the price of country 1 (the low-price country) spread to country 2 (the high-price one), as soon as country 2 can observe it. If country 1 approves the drug before country 2 ($m_1 + 1 \leq m_2$), then the firm sells the drug at price w_1 in both countries from their respective ATs onwards. The price w_1 is the highest possible one allowed by IRP when the firm sells in both countries and $m_1 + 1 \leq m_2$. If instead, $m_1 + 1 \geq m_2$, then the price w_1 spreads to country 2 one period after approval in country 1.

If the firm sells the drug in the high-price country only, from its AT onwards, its intertemporal profits are π_2, with

$$\pi_2 = \frac{\beta^{m_2}}{1-\beta} w_2 \omega_2.$$

IRP is $a fortiori$ inactive when the firm sells in country 2 only and the firm sets a price equal to the WTP in country 2 so as to fully extract the country's surplus.

To illustrate the effect of different ATs on the firm's optimal strategy, we simply compare the firm's gains π_{12} and π_2 and establish the following firm's incentives. The firm is better off selling in both countries 1 and 2 from their respective ATs onwards if and only if

$$\pi_{12} \geq \pi_2 \iff \beta^{Max(m_2-m_1,0)} \leq \frac{w_1}{w_2-w_1}\frac{\omega_1}{\omega_2}.$$

Otherwise, the firm is better off selling in the high-price country only. Concretely, the firm is more likely to commercialize its new drug in both countries if the high-price country approves the drug sufficiently later than the low-price country (i.e., $m_2 - m_1$ is sufficiently high); if the firm is not patient enough regarding future profits (i.e., if β is sufficiently low); and the market in the low-price country is profitable enough compared to the one in country 2 (i.e., $w_1\omega_1$ is high relative to $w_2\omega_2$).

In other words, if approval times are similar across countries, the firm's gains from selling in country 1 may not be worth the losses from spreading the low price of country 1 to country 2. Conversely, if the low-price country approves the new drug sufficiently before the high-price country, then it becomes optimal for the firm to sell in both countries as soon as the market authorization is delivered. The basic trade-off between sales in one additional market against the spreading of a low price through IRPs is modified in favor of the country with an early market authorization. The reason here is that the gains from selling in an additional market concern a higher number of periods while the losses from spreading a low price through IRPs does not vary. The arguments developed so far are formally extended to a more-general n-country setting in Sections 4 and 5.

The firm's profits π_{12} and π_2 are always decreasing in the approval times. Therefore, the firm is better off when countries approve the drug as soon as possible, no matter whether the firm commercializes it in both countries or not.

This two-country setting allows discussing the countries welfare as a function of ATs. Anticipating the firm's incentives analyzed so far, we can write the countries' intertemporal surplus as follows.

$$s_1 = 0;$$

and

$$s_2 = \begin{cases} 0 & \text{, if } \beta^{Max(m_2-m_1,0)} > \frac{w_1}{w_2-w_1}\frac{\omega_1}{\omega_2} \\ \beta^{Max(m_1+1,m_2)}(w_2-w_1)\omega_2 & \text{, if } \beta^{Max(m_2-m_1,0)} \leq \frac{w_1}{w_2-w_1}\frac{\omega_1}{\omega_2} \end{cases}.$$

The surplus in country 1 is null no matter whether the firm commercializes the drug there or not. In fact, the low-price country would pay exactly its WTP if the drug is available since the firm uses its monopolistic position to extract any positive surplus from the country. So, the low-price country is indifferent between approval procedures since its surplus is invariant to ATs.

The surplus in country 2 is positive as soon as the firm sells the new drug in country 1 as well. In this case, the high-price country enjoys a price w_1 lower than its WTP w_2. Conversely, if the firm sells the drug in the high-price country only, it does so at a price equal to its WTP to extract any positive surplus from the country.

Let us also notice that s_2 reaches its maximum value when

$$\beta^{Max(m_2-m_1,0)} = \frac{w_1}{w_2-w_1}\frac{\omega_1}{\omega_2}.$$

Said differently, the best reply of country 2 in terms of AT to country 1's AT is as follows:

$$m_2^*(m_1) = m_1 + \frac{ln\frac{w_1\omega_1}{(w_2-w_1)\omega_2}}{ln\beta}.$$

With such a time spam between ATs, the firm has an incentive to launch the new drug in both countries. Any lower m_2 would make the firm forego its sales in the low-price country and the high-price country would not enjoy the benefits of a low reference price. Instead, any higher m_2 would make country 2 forego positive surplus during some periods of time since the low-price country is served already. This reasoning rests on the ability of countries to commit to pre-determinate ATs. National legal limits on the length of time dedicated to examine drug approval applications can help such a commitment.

To conclude, the high-price country best strategy is to approve the drug sufficiently later than the low-price country. As such, the high-price country should prefer a non-centralized approval procedure, which allows non-simultaneous ATs, while the low-price country is indifferent.

4 General results on optimal price vectors

The purpose of our study is to draw some conclusions about the effect of ATs on the set of countries in which the drug will be launched. In order to do that, we first need to derive some properties about the price vectors.

The following proposition is a generalization in the case with different ATs of a result already stated in [Houy and Jelovac, 2015]. It states that when the drug is launched by the seller in a country, it is never withdrawn from this country afterward.

Proposition 1
Let $P \in P^a$. $\forall t \in \mathbb{N}, S^t(P) \subseteq S^{t+1}(P)$.

Intuitively, for the seller, the effect of launching the drug in a country is to sell more at the potential cost of selling at a lower price in other countries. Since the IRPs consider all past prices, selling at a point in time in a country already sets a reference price for the future. Hence, withdrawing the drug in the future consists only in losing MS.

We can define, for a given optimal price vector $P \in P^a$, $\overline{S(P)}$ as the set of countries in which the drug is launched at some point in time. Formally,

Definition 2

$\forall P \in P^a$, $\overline{S(P)} = \{i \in N, \exists t \in \mathbb{N}, i \in S^t(P)\}$.

Because the set of countries in which the drug is launched can only grow larger in time, and because the set of countries we consider is finite, there exists a point in time after which, $\overline{S(P)}$ is exactly the set of countries in which the drug is ever sold.

The following proposition states that there is monotonicity in the set of countries in which the drug is sold at each period. Hence, if the drug is sold in period t in country i and if country j's WTP is greater that country i's, then the drug should also be sold in country j. Of course, this result is conditional on the approval process to be completed in country j.

Proposition 2

Let $P \in P^a$. Let $i \in N, t \in \mathbb{N}$ be such that $i \in S^t(P)$. Let $j \in N$ be such that $w_i \leq w_j$ and $m_j \leq t$. Then, $j \in S^t(P)$.

Intuitively, if the drug is sold in country i with a lower WTP than country j's, then, setting the same price in country j as in country i implies selling larger quantities (after j's AT) without having any effect on future sales and prices through IRPs.

Finally, the following proposition states that for some optimal price vector and for any period t, there is no entry in any market in period t if no country has an AT exactly equal to t. Then, any launching period corresponds to an AT in a country. Notice that, possibly, launching can occur in country i when a market becomes available in country $j \neq i$ (*i.e* at j's AT).

Proposition 3

$\exists P \in P^a, \forall t \in \mathbb{N}, [\{i \in N, m_i = t\} = \emptyset \textit{ and } t > 0] \Rightarrow S^{t-1}(P) = S^t(P)$.

Notice that Proposition 3 states that the condition regarding ATs and launching periods is imposed for at least one but not all optimal price vectors. The reason is the following. Consider for the sake of the illustration the following example.

Example 1

Let us consider 2 countries, $N = (1, 2)$ with WTPs $(1, 2)$, MSs $(9, 10)$ and ATs $(0, 0)$. Let $\beta = 0.9$.

It is straightforward, following Proposition 3 to check that the following price vector $P = (p_i^t)_{i \in N, t \in \mathbb{N}} \in \mathcal{P}$ is optimal if $\forall i \in N, \forall t \in \mathbb{N}, p_i^t = 2$. In this case, $\forall t \in \mathbb{N}, S^t(P) = \{2\}$ and $\Pi(P) = 200$. However, let us have $P' = ({p'}_i^t)_{i \in N, t \in \mathbb{N}} \in \mathcal{P}$ defined by

$$\forall i \in N, \forall t \in \mathbb{N}, {p'}_i^t = \begin{cases} 2 & \text{, if } t = 0 \\ 2 & \text{, if } t = 1 \text{ and } i = 2 \\ 1 & \text{, otherwise} \end{cases} .$$

It is straightforward to check that $\Pi(P') = 200$. Hence, P' is also optimal with $S^0(P) = \{2\}$ and $\forall t > 0 \in \mathbb{N}, S^t(P) = \{1, 2\}$. In this case, the conditions stated in Proposition 3 are not satisfied for P'. The reason is the following. Example 1 describes a very particular situation where the loss suffered by the seller in terms of price decreasing in country 2 in order to sell in country 1 is exactly equal to the surplus profit earned from increasing the MS. In this particular case, the launching in country 1 can be implemented at any point in time without losing profit and hence is not conditional on the AT in another country. These are the only cases for which Proposition 3 does not generalize to all optimal price vectors.

5 The effect of Approval Times

We will now derive some results regarding the sets of countries in which the drug is ever launched and the effect of ATs on this set. First, let us consider the sets of countries in which the drug is ever launched when all ATs are null and the seller is constrained to launch the drug in the same countries at each period. Formally, if $\mathcal{S}^0$ is the set of these sets, we have,

Definition 3

$$\mathcal{S}^0 = \arg\max_{S \subseteq N} \sum_{i \in S} w_i \omega_i + \frac{\beta}{1-\beta} \left(\min_{i \in S} w_i \right) \left(\sum_{i \in S} \omega_i \right).$$

The following two propositions show that $\mathcal{S}^0$ is exactly the set of sets of countries in which the drug is ever launched when all ATs are null. Then, the constraint that the drug should be launched in the same countries at each period that we considered in order to define $\mathcal{S}^0$ is not a binding one.

Proposition 4

Assume $\forall i \in N, m_i = 0$. $\forall S \in \mathcal{S}^0, \exists P \in P^a, \overline{S(P)} = S$.

Proposition 5

Assume $\forall i \in N, m_i = 0$. $\forall P \in P^a, \overline{S(P)} \in \mathcal{S}^0$.

The intuition is the same as the one we gave in Example 1. As we stated, in the case where all ATs are null, the only reason why we could have a launch in a country after period 0 is when the decrease in price it induces for the sales in the other countries is exactly compensated by the gain in MS. But then, it is also optimal for the seller to launch the drug in this country from period 0.

Proposition 6 shows that when ATs are not necessarily all null, the set of countries in which the drug is sold can only be larger than in the case where all ATs are null. Said differently, differences in ATs increase the set of countries in which a new drug is launched.

Proposition 6

$\forall P \in P^a$, $\exists S \in \mathcal{S}^0, S \subseteq \overline{S(P)}$.

The following two propositions generalize Propositions 4 and 5 when all the ATs are either null or unity. Then, $\mathcal{S}^0$ is exactly the set of sets of countries in which the drug is ever launched when all ATs are null or when they are all 0 or 1. Including a 1 period delay in the ATs for some countries has no effect on which countries the seller will choose optimally to ever launch the drug. The reason is that it takes exactly one period of time for prices to spread into the countries where they ever spread when the IRPs are complete.

Proposition 7

Assume $\{t \in \mathbb{N}, \exists i \in N, m_i = t\} = \{0, 1\}$. $\forall P \in P^a, \overline{S(P)} \in \mathcal{S}^0$.

Proposition 8

Assume $\{t \in \mathbb{N}, \exists i \in N, m_i = t\} = \{0, 1\}$. $\forall S \in \mathcal{S}^0, \exists P \in P^a, \overline{S(P)} = S$.

6 Conclusion

This paper analyzes the influence of drug approval procedures on the access to new drugs in different countries when prices are regulated according to an international reference pricing policy.

We confirm the main result of [Houy and Jelovac, 2015], for drug approvals that are close to each other in time. Concretely, when all drug approvals are either simultaneous or one period of time different, a pharmaceutical firm has no incentive to sequentially launch its new drugs in different countries. Instead, it optimally launches its products in high-WTP countries as soon as they are approved there. It never launches drugs in other countries to avoid spreading a low price to the high-WTP countries through IRPs.

However, as the time differences between drug approvals increase above one period of time, the conclusions drastically change. The firm can face different incentives when deciding whether to launch a new drug in a country or not. In particular, if a low-WTP country delivers a drug approval early enough compared to high-WTP countries, the firm can be better-off commercializing the drug in the low-WTP countries too, even if it spreads a low price to other countries through IRPs. Moreover, we show that the number of countries in which the firm optimally commercializes a new drug is higher when the differences between drug approval times exceeds one period of time.

In a less general two-country setting, we also show that a high-price country's best strategy is to approve the drug sufficiently later than a low-price country. This strategy ensures that the low-price market is served and the high-price country can enjoy a low reference price by IRP. As such, a high-price country should prefer a non-centralized approval procedure, which allows non-simultaneous ATs, while a low-price country is indifferent. This result is based on the countries ability to commit to predetermined ATs, which can be eased by the legal deadlines for examining applications for drug approval. The firm in turn is better off when countries approve the drug as soon as possible, no matter whether the firm commercializes it in both countries or not.

An interesting extension of this analysis would consider a finite time horizon instead of an infinite one. It would respect the finite period during which a pharmaceutical producer enjoys a monopolistic position under patent protection. However, the differences in WTP between countries do not vanish with the expiration of a patent and the implications of IRP may be persistent, even tough the monopolistic position of the producer weakens.

Another issue that we disregard here is that firms decide when to fill an application for drug approval. We can expect a correlation between approval times, decided by countries, and application times, decided by the pharmaceutical firms. Our results so far do not allow any conjecture about the strategic timing of applications by firms.

We can also think of another extension for future research if relaxing an implicit assumption of our model. We considered that the drug approvals are constraints for pharmaceutical firms only because they impose delays in the launching sequence. Obviously, there also exists a risk dimension in this applied problem. Indeed, submitting a file for approval in a country is risky and the outcome can always be negative. If we consider that this risk is independent across countries and make the time decision to seek approval an endogenous variable, the optimal approval application strategy is obviously to apply for an approval in all countries as soon as possible since this strategy is one that leads to the least binding constraints for the launching sequence. Then, in this setting, our assumption of exogenous application timing has no strength. However, if we consider more realistically that the approval decision risk is dependent across countries, then, the application sequence is not trivial and interferes with the launching sequence.

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A Proofs

Let us set the following definitions:

- let $\overline{w} = \max_{i \in N} w_i$ be the largest WTP for all countries,
- let $\overline{m} = \max_{i \in N} m_i$ be the maximum AT for all countries,
- let $\forall A \subseteq N, w_A = \min_{i \in A} w_i$ be the minimum WTP for the subset of countries in A.

Lemma 1

Let $P \in P^a$. $\exists t \in \mathbb{N}, S^t(P) \neq \emptyset$.

Proof of Lemma 1: Let us have $\epsilon \in \mathbb{R}$ such that $0 < \epsilon < w_N$. Let us define $P' = ({p'}_i^t)_{i \in N, t \in \mathbb{N}} \in \mathcal{P}$ be such that $\forall i \in N, \forall t \in \mathbb{N}, {p'}_i^t = \epsilon$. By definition, $\forall i \in N, \forall t \geq m_i, i \in S^t(P')$ and then $\Pi(P') > 0$. By definition, $\forall P \in P^a, \Pi(P) \geq \Pi(P') > 0$. $\square$

Lemma 2

Let $P = (p_i^t)_{i \in N, t \in \mathbb{N}} \in P^a$. $\forall t \in \mathbb{N}, \forall i \in N, p_i^t > 0$.

Proof of Lemma 2: Assume it is not the case. Let $T \in \mathbb{N}$ be such that $\exists i \in N, p_i^T = 0$.

Let us have $\epsilon \in \mathbb{R}^{+*}$ such that $0 < \epsilon < w_N$. Let $P' = ({p'}_{i'}^{t'})_{i' \in N, t' \in \mathbb{N}} \in \mathcal{P}$ be defined as:

$$\forall i' \in N, \forall t' \in \mathbb{N}, {p'}_{i'}^{t'} = \begin{cases} \epsilon & \text{, if } p_{i'}^{t'} < \epsilon \\ p_{i'}^{t'} & \text{, otherwise} \end{cases} .$$

Let $i \in S^0(P)$. By definition, $p_i^t \leq w_i$. Then, since $\epsilon < w_N$, ${p'}_i^t \leq w_i$ which implies $i \in S^0(P')$.

Let $i \in S^t(P)$ with $t > 0$. By definition, $p_i^t \leq w_i$ and $p_i^t \leq \min_{i' \in N, t' < t} p_{i'}^{t'}$. Then, since $\epsilon < w_N$, ${p'}_i^t \leq w_i$. Moreover, it is straightforward to check that ${p'}_i^t \leq \min_{i' \in N, t' < t} {p'}_i^t$. Hence, $i \in S^t(P')$.

$\Pi(P) = \sum_{t \leq T} \beta^t \sum_{i \in S^t(P)} p_i^t \omega_i + \sum_{t > T} \beta^t \sum_{i \in S^t(P)} p_i^t \omega_i$. By what we showed, $\sum_{t \leq T} \beta^t \sum_{i \in S^t(P)} p_i^t \omega_i \leq \sum_{t \leq T} \beta^t \sum_{i \in S^t(P')} {p'}_i^t \omega_i$ and it is straightforward to check that $\sum_{t > T} \beta^t \sum_{i \in S^t(P)} p_i^t \omega_i = 0 < \sum_{t > T} \beta^t \sum_{i \in S^t(P')} {p'}_i^t \omega_i$.

Then, $\Pi(P') > \Pi(P)$ which contradicts the assumption that $P \in P^a$. $\Box$

Definition 4

Let $P = (p_i^t)_{i\in N, t\in\mathbb{N}} \in \mathcal{P}$, $MP_i^t(P) \in \mathbb{R}^+$ is defined as follows

$$MP_i^t(P) = \begin{cases} \min(w_i, \min_{i'\in N, t'<t} p_{i'}^{t'}) & \text{, if } t > 0 \\ w_i & \text{, if } t = 0 \end{cases}.$$

Proof of Proposition 1: Assume it is not the case. Let $i \in N, t \in \mathbb{N}$ be such that $i \in S^t(P)$ and $i \notin S^{t+1}(P)$. Let $P' = (p'^{t'}_{i'})_{i'\in N, t'\in\mathbb{N}} \in \mathcal{P}$ be defined as:

$$\forall i' \in N, \forall t' \in \mathbb{N}, p'^{t'}_{i'} = \begin{cases} \min_{i''\in N, t''<t+1} p_{t''}^{i''} & \text{, if } i' = i \text{ and } t' = t+1 \\ p_{i'}^{t'} & \text{, otherwise} \end{cases}.$$

By definition, $\forall i' \in N, \forall t' \leq t+1, MP_{i'}^{t'}(P) = MP_{i'}^{t'}(P')$.

$i \in S^t(P)$ implies $p_i^t \leq w_i$ and $t \geq m_i$. Hence, $i \notin S^{t+1}(P)$ implies $p_i^{t+1} > \min_{i''\in N, t''<t+1} p_{t''}^{i''} = p'^{t+1}_i$. Then $\forall t' \geq t+1 \in \mathbb{N}, \forall i' \in N, p_{i'}^{t'} \geq p'^{t'}_{i'}$. Then, $\forall i' \in N, \forall t' > t+1, MP_{i'}^{t'}(P) \geq MP_{i'}^{t'}(P')$.

Moreover, $\forall t' > t+1$,

$$\min_{i''\in N, t''<t'} p'^{t''}_{i''} =$$
$$\min(\min_{i''\in N, t''<t+1} p'^{t''}_{i''}, \min_{i''\in N} p'^{t+1}_{i''}, \min_{i''\in N, t'>t''>t+1} p'^{t''}_{i''}) =$$
$$\min(\min_{i''\in N, t''<t+1} p_{i''}^{t''}, \min_{i''\in N} p'^{t+1}_{i''}, \min_{i''\in N, t'>t''>t+1} p_{i''}^{t''}) =$$
$$\min(\min_{i''\in N, t''<t+1} p_{i''}^{t''}, \min_{i''\in N\setminus\{i\}} p'^{t+1}_{i''}, p'^{t+1}_i, \min_{i''\in N, t'>t''>t+1} p_{i''}^{t''}) =$$
$$\min(\min_{i''\in N, t''<t+1} p_{i''}^{t''}, \min_{i''\in N\setminus\{i\}} p_{i''}^{t+1}, \min_{i''\in N, t'>t''>t+1} p_{i''}^{t''}) \geq$$
$$\min_{i''\in N, t''<t'} p_{i''}^{t''}.$$

Hence, $\forall i' \in N, \forall t' > t+1, MP_{i'}^{t'}(P) \leq MP_{i'}^{t'}(P')$.

Then, $\forall i' \in N, \forall t' > t+1, MP_{i'}^{t'}(P) = MP_{i'}^{t'}(P')$.

Then, $\forall i' \in N, \forall t' \in \mathbb{N}, MP_{i'}^{t'}(P) = MP_{i'}^{t'}(P')$. Hence, $\forall i' \in N, \forall t' \in \mathbb{N}, [i' \neq i \text{ or } t' \neq t] \Rightarrow [i' \in S^{t'}(P') \Leftrightarrow i' \in S^{t'}(P)]$.

$i \in S^t(P)$ implies $p_i^t \leq w_i$ and $t \geq m_i$. $p_i^t \leq w_i$ implies ${p'}_i^{t+1} \leq w_i$. Then, by definition, ${p'}_i^{t+1} \leq MP_i^{t+1}(P')$. Then, $i \in S^{t+1}(P')$.

Then, by Lemma 2, $\Pi(P') > \Pi(P)$ which contradicts the fact that $P \in P^a$. □

Lemma 3

Let $P = (p_i^t)_{i \in N, t \in \mathbb{N}} \in P^a$. $\forall i \in S^t(P), \forall t \in \mathbb{N}, p_i^t = MP_i^t(P)$.

Proof of Lemma 3: Assume it is not the case. Let $i \in N$ and $t \in \mathbb{N}$ be such that $p_i^t \neq MP_i^t(P)$. By definition, if $p_i^t > MP_i^t(P)$, then, $i \notin S^t(P)$ contradicting the assumptions. Then, assume $p_i^t < MP_i^t(P)$. Let $P' = ({p'}_{i'}^{t'})_{i' \in N, t' \in \mathbb{N}} \in \mathcal{P}$ be defined as:

$$\forall i' \in N, \forall t' \in \mathbb{N}, {p'}_{i'}^{t'} = \begin{cases} MP_i^t(P) & \text{, if } i' = i \text{ and } t' = t \\ p_{i'}^{t'} & \text{, otherwise} \end{cases}.$$

By definition, since we assumed $p_i^t < MP_i^t(P) = {p'}_i^t$, $\forall i' \in N, \forall t' \in \mathbb{N}, MP_{i'}^{t'}(P) \leq MP_{i'}^{t'}(P')$ and hence, $\forall i' \in N, \forall t' \in \mathbb{N}, [i' \neq i \text{ or } t' \neq t] \Rightarrow [i' \in S^{t'}(P) \Rightarrow i' \in S^{t'}(P')]$.

By definition, $i \in S^t(P)$ implies $t \geq m_i$. Besides, by definition of P', ${p'}_i^t \leq MP_i^t(P)$. Hence, $i \in S^{t'}(P')$.

Hence, it is straightforward to check that $\Pi(P') > \Pi(P)$, contradicting the assumption stating $P \in P^a$. □

Lemma 4

Let $P = (p_i^t)_{i \in N, t \in \mathbb{N}} \in P^a$. $\forall i \in S^0(P), p_i^0 = w_i$.

Proof of Lemma 4: By Lemma 3. □

Proof of Proposition 2: Assume it is not the case, *i.e.* $j \notin S^t(P)$. Let $P' = ({p'}_{i'}^{t'})_{i' \in N, t' \in \mathbb{N}} \in \mathcal{P}$ be defined as:

$$\forall i' \in N, \forall t' \in \mathbb{N}, {p'}_{i'}^{t'} = \begin{cases} MP_j^t(P) & \text{, if } i' = j \text{ and } t' = t \\ p_{i'}^{t'} & \text{, otherwise} \end{cases}.$$

By definition, $\forall i' \in N, \forall t' \leq t, MP_{i'}^{t'}(P) = MP_{i'}^{t'}(P')$.

Moreover, since by assumption, $m_j \leq t$, $j \notin S^t(P)$ implies $p_j^t > MP_j^t(P)$. Hence, $p_j^t > {p'}_j^t$. Then, by definition, $\forall i' \in N, \forall t' > t, MP_{i'}^{t'}(P) \geq MP_{i'}^{t'}(P')$.

Moreover, $\forall t' > t$,

$$\min_{i'' \in N, t'' < t'} {p'}_{i''}^{t''} =$$
$$\min(\min_{i'' \in N, t'' < t} {p'}_{i''}^{t''}, \min_{i'' \in N} {p'}_{i''}^{t}, \min_{i'' \in N, t' > t'' > t} {p'}_{i''}^{t''}) =$$
$$\min(\min_{i'' \in N, t'' < t} p_{i''}^{t''}, \min_{i'' \in N} {p'}_{i''}^{t}, \min_{i'' \in N, t' > t'' > t} p_{i''}^{t''}) =$$
$$\min(\min_{i'' \in N, t'' < t} p_{i''}^{t''}, \min_{i'' \in N \setminus \{i,j\}} {p'}_{i''}^{t}, {p'}_j^t, {p'}_i^t, \min_{i'' \in N, t' > t'' > t+1} p_{i''}^{t''}) =$$
$$\min(\min_{i'' \in N, t'' < t} p_{i''}^{t''}, \min_{i'' \in N \setminus \{i,j\}} p_{i''}^{t}, {p'}_j^t, p_i^t, \min_{i'' \in N, t' > t'' > t+1} p_{i''}^{t''}).$$

However, since $w_i \leq w_j$, $MP_i^t(P) \leq MP_j^t(P)$. By Lemma 3, since $i \in S^t(P)$, $p_i^t = MP_i^t(P)$. Then, $p_i^t = MP_i^t(P) \leq MP_j^t(P) = {p'}_j^t$. Then,

$$\min_{i'' \in N, t'' < t'} {p'}_{i''}^{t''} =$$
$$\min(\min_{i'' \in N, t'' < t} p_{i''}^{t''}, \min_{i'' \in N \setminus \{i,j\}} p_{i''}^{t}, p_i^t, \min_{i'' \in N, t' > t'' > t+1} p_{i''}^{t''}) \geq$$
$$\min_{i'' \in N, t'' < t'} p_{i''}^{t''}.$$

Hence, $\forall i' \in N, \forall t' > t, MP_{i'}^{t'}(P) \leq MP_{i'}^{t'}(P')$.

Then, $\forall i' \in N, \forall t' > t, MP_{i'}^{t'}(P) = MP_{i'}^{t'}(P')$. Hence, $\forall i' \in N, \forall t' \in \mathbb{N}, [i' \neq j \text{ or } t' \neq t] \Rightarrow [i' \in S^{t'}(P') \Leftrightarrow i' \in S^{t'}(P)]$.

Since by assumption, $t \geq m_j$ and by definition of P', ${p'}_j^t \leq MP_j^t(P) = MP_j^t(P')$, then, $j \in S^t(P')$.

Then, using Lemma 2, $\Pi(P') > \Pi(P)$ which contradicts the fact that $P \in P^a$. $\square$

In the following, we will use an arbitrary strictly positive number $\delta \in \mathbb{R}^{+*}$.

Definition 5

Let $P = (p_i^t)_{i \in N, t \in \mathbb{N}} \in P^a$. $\tilde{P} = (\tilde{p}_i^t)_{i \in N, t \in \mathbb{N}} \in \mathcal{P}$ is defined as:

$$\forall i' \in N, \forall t' \in \mathbb{N}, \tilde{p}_{i'}^{t'} = \begin{cases} p_{i'}^{t'} & , \text{ if } i' \in S^{t'}(P) \\ \overline{w} + \delta & , \text{ if } i' \notin S^{t'}(P) \end{cases}.$$

Lemma 5

Let $\forall P \in P^a, \forall t \in \mathbb{N}, S^t(\tilde{P}) = S^t(P)$.

Proof of Lemma 5: Let $P = (p_i^t)_{i\in N, t\in\mathbb{N}} \in P^a$. By definition, $\tilde{P} = (\tilde{p}_{i'}^{t'})_{i'\in N, t'\in\mathbb{N}} \in \mathcal{P}$ satisfies

$$\forall i' \in N, \forall t' \in \mathbb{N}, \tilde{p}_{i'}^{t'} = \begin{cases} p_{i'}^{t'} & \text{, if } i' \in S^{t'}(P) \\ \overline{w} + \delta & \text{, if } i' \notin S^{t'}(P) \end{cases} .$$

Let $i \in N, t \in \mathbb{N}$ such that $i \in S^t(P)$. Then, by definition, $t \geq m_i$, $p_i^t \leq w_i$ and hence $\tilde{p}_i^t \leq w_i$. Then, if $t = 0$, $i \in S^t(\tilde{P})$. Assume $t > 0$. $i \in S^t(P)$ implies

$$p_i^t = \tilde{p}_i^t \leq \min_{i' \in N, t' < t}(p_{i'}^{t'}, w_i)$$

$$\tilde{p}_i^t \leq \min_{t'<t}(\min_{i' \in S^{t'}(P)} p_{i'}^{t'}, \min_{i' \notin S^{t'}(P)} p_{i'}^{t'}, w_i) = \min_{t'<t}(\min_{i' \in S^{t'}(P)} \tilde{p}_{i'}^{t'}, \min_{i' \notin S^{t'}(P)} p_{i'}^{t'}, w_i)$$

$$\tilde{p}_i^t \leq \min_{t'<t}(\min_{i' \in S^{t'}(P)} \tilde{p}_{i'}^{t'}, w_i) = \min_{t'<t}(\min_{i' \in S^{t'}(P)} \tilde{p}_{i'}^{t'}, \overline{w} + \delta, w_i)$$

$$\tilde{p}_i^t \leq \min_{t'<t, i' \in N}(\tilde{p}_{i'}^{t'}, w_i)$$

Then, since $t \geq m_i$, $i \in S^t(\tilde{P})$.

Let $i \in N, t \in \mathbb{N}$ be such that $i \notin S^t(P)$. Then, by definition, $\tilde{p}_i^t = \overline{w} + \delta > w_i$ which implies $i \notin S^t(\tilde{P})$. $\square$

Lemma 6

Let $\forall P \in P^a, \tilde{P} \in P^a$.

Proof of Lemma 6: Follows directly from the definition of $\tilde{P}$ and Lemma 5. $\square$

Lemma 7

$\forall P = (p_i^t)_{i\in N, t\in\mathbb{N}} \in P^a, \forall i \in N, \forall t \in \mathbb{N}$,

1. $t = 0 \Rightarrow [i \in S^t(P) \Rightarrow p_i^t = w_i]$,

2. $t > 0 \Rightarrow [i \in S^t(P) \Rightarrow p_i^t = \min(w_i, \min_{t'<t, i' \in S^{t'}(P)} w_{i'})]$.

Proof of Lemma 7: 1 follows directly from Lemma 3.

Let us prove 2. By Lemma 6, $\tilde{P} \in P^a$. Then, by Lemma 3, $\forall t \in \mathbb{N}, \forall i \in N, i \in S^t(\tilde{P}) \Rightarrow \tilde{p}_i^t = MP_i^t(\tilde{P})$. By recursivity, it is straightforward to check that $\forall t \in \mathbb{N}, \forall i \in N, MP_i^t(\tilde{P}) = \min(w_i, \min_{t'<t, i' \in S^{t'}(\tilde{P})} w_{i'})$. By Lemma 5, $\forall t \in \mathbb{N}, S^t(\tilde{P}) = S^t(P)$. Moreover, by definition, $\forall t \in \mathbb{N}, \forall i \in N, i \in S^t(P) \Rightarrow p_i^t = \tilde{p}_i^t$. Then, $\forall t \in \mathbb{N}, \forall i \in N, i \in S^t(P) \Rightarrow p_t^i = \tilde{p}_i^t = MP_i^t(\tilde{P}) = \min(w_i, \min_{t'<t, i' \in S^{t'}(P)} w_{i'})$. □

Lemma 8

$\exists P = (p_i^t)_{i \in N, t \in \mathbb{N}} \in P^a, \forall i \in N, \forall t \in \mathbb{N}$,

1. $i \notin S^t(P) \Rightarrow p_i^t = \overline{w} + \delta$,
2. $t = 0 \Rightarrow [i \in S^t(P) \Rightarrow p_i^t = w_i]$,
3. $t > 0 \Rightarrow [i \in S^t(P) \Rightarrow p_i^t = \min(w_i, \min_{t'<t, i' \in S^{t'}(P)} w_{i'})]$.

Proof of Lemma 8: Let $P \in P^a$. By Lemma 6, $\tilde{P} \in P^a$. Then, by Lemma 7, $\tilde{P}$ satisfies 2 and 3. Moreover, by definition, $\tilde{P}$ satisfies 1. □

Definition 6

Let $P^* \subseteq P^a$ be the subset of optimal price vectors such that $\forall P \in P^*$,

1. $i \notin S^t(P) \Rightarrow p_i^t = \overline{w} + \delta$,
2. $t = 0 \Rightarrow [i \in S^t(P) \Rightarrow p_i^t = w_i]$,
3. $t > 0 \Rightarrow [i \in S^t(P) \Rightarrow p_i^t = \min(w_i, \min_{t'<t, i' \in S^{t'}(P)} w_{i'})]$.

Lemma 9

$P^* \neq \emptyset$.

Proof of Lemma 9: By Lemma 8. □

Lemma 10

$\forall P \in P^*,\ \exists P' \in P^*, \forall t \in \mathbb{N}, t > \overline{m} \Rightarrow S^{t-1}(P') = S^t(P') = \overline{S(P)}$.

Proof of Lemma 10: Let $P \in P^* \subseteq P^a$. By definition, P satisfies $\forall i \in N, \forall t \in \mathbb{N}$,

- $i \notin S^t(P) \Rightarrow p_i^t = \overline{w} + \delta$,
- $t = 0 \Rightarrow [i \in S^t(P) \Rightarrow p_i^t = w_i]$,
- $t > 0 \Rightarrow [i \in S^t(P) \Rightarrow p_i^t = \min(w_i, \min_{t'<t, i' \in S^{t'}(P)} w_{i'})]$.

Let $T > \overline{m}$ be the latest period for which $S^{T-1}(P) \neq S^T(P)$. If T is not defined, the proof is complete. By Proposition 1, $S^{T-1}(P) \subset S^T(P)$. Let us define $A = S^{T-1}(P) \setminus S^T(P) \neq \emptyset$. Let $P' = ({p'}_{i'}^{t'})_{i' \in N, t' \in \mathbb{N}} \in \mathcal{P}$ be defined as:

$$\forall i' \in N, \forall t' \in \mathbb{N}, {p'}_{i'}^{t'} = \begin{cases} w_{i'} & \text{, if } i' \in A \text{ and } t' = T-1 \\ \min_{i \in S^T(P)} w_i & \text{, if } i' \in S^T(P) \text{ and } t' = T \\ p_{i'}^{t'} & \text{, otherwise} \end{cases}.$$

It is straightforward to check that

$$\begin{aligned} &\Pi(P) - \Pi(P') = \\ &(\beta^T - \beta^{T-1}) \left(\sum_{i \in A} w_i \omega_i \right) \\ &+\beta^T \left(w_{S^{T-1}(P)} - w_{S^T(P)} \right) \sum_{i \in S^{T-1}(P)} \omega_i \\ &-\beta^T w_{S^T(P)} \sum_{i \in A} \omega_i. \end{aligned}$$

Moreover, $P \in P^a$ implies $\Pi(P) - \Pi(P') \geq 0$. Then,

$$\sum_{i \in A} w_i \omega_i \leq \frac{\beta}{1-\beta} \left[\left(w_{S^{T-1}(P)} - w_{S^T(P)} \right) \left(\sum_{i \in S^{T-1}(P)} \omega_i \right) - w_{S^T(P)} \left(\sum_{i \in A} \omega_i \right) \right]. \tag{1}$$

Now, let $P'' = (p''^{t'}_{i'})_{i' \in N, t' \in \mathbb{N}} \in \mathcal{P}$ be defined as:

$$\forall i' \in N, \forall t' \in \mathbb{N}, p''^{t'}_{i'} = \begin{cases} \overline{w} + \delta & \text{, if } i' \in A \text{ and } t' \geq T \\ w_{S^{T-1}(P)} & \text{, if } i' \in S^{T-1}(P) \text{ and } t' \geq T+1 \\ p^{t'}_{i'} & \text{, otherwise} \end{cases} .$$

It is straightforward to check that

$$\Pi(P) - \Pi(P'') =$$

$$\beta^T \left(\sum_{i \in A} w_i \omega_i \right)$$

$$+\frac{\beta^{T+1}}{1-\beta} \left(\sum_{i \in S^T(P)} w_{S^T(P)} \omega_i - \sum_{i \in S^{T-1}(P)} w_{S^{T-1}(P)} \omega_i \right)$$

Moreover, $P \in P^a$ implies $\Pi(P) - \Pi(P'') \geq 0$. Then,

$$\sum_{i \in A} w_i \omega_i \geq \frac{\beta}{1-\beta} \left[\left(w_{S^{T-1}(P)} - w_{S^T(P)} \right) \left(\sum_{i \in S^{T-1}(P)} \omega_i \right) - w_{S^T(P)} \left(\sum_{i \in A} \omega_i \right) \right] . \tag{2}$$

Using Equations 1 and 2, we have

$$\sum_{i \in A} w_i \omega_i = \frac{\beta}{1-\beta} \left[\left(w_{S^{T-1}(P)} - w_{S^T(P)} \right) \left(\sum_{i \in S^{T-1}(P)} \omega_i \right) - w_{S^T(P)} \left(\sum_{i \in A} \omega_i \right) \right] .$$

And then, $\Pi(P) = \Pi(P')$. Then, $P' \in P^a$. Moreover, it is straightforward to check that $P' \in P^*$ and $\overline{S(P')} = \overline{S(P)}$. Let $T' > \overline{m}$ be the latest period for which $S^{T-1}(P') \neq S^T(P')$. If T' is not defined, the proof is complete. If $T' < T$ is defined, repeat the previous steps of the proof a finite number of times until T' is not defined. □

Lemma 11

$\exists P \in P^*, \forall t \in \mathbb{N}, [\{i \in N, m_i = t\} = \emptyset \textit{ and } t > 0] \Rightarrow S^{t-1}(P) = S^t(P).$

Proof of Lemma 11: Let us define $P^{**} = \{P \in P^*, \forall t > \overline{m}, S^{t-1}(P) = S^t(P)\}$. By Lemmas 9 and 10, $P^{**} \neq \emptyset$. Assume that $\forall P \in P^{**}, \exists t < \overline{m}, [\{i \in N, m_i = t\} = \emptyset \text{ and } S^{t-1}(P) \neq S^t(P)]$. $\forall P \in P^{**}$, let $\#T(P) = \#\{t < \overline{m}, [\{i \in N, m_i = t\} = \emptyset \text{ and } S^{t-1}(P) \neq S^t(P)]\}$. By assumption, $\forall P \in P^{**}, \#T(P) \geq 1$. Moreover, by since $\overline{m}$ is finite, $\forall P \in P^{**}$, $\#T(P)$ is finite. By Proposition 1, $\forall P \in P^{**}, \forall t \in \mathbb{N}, S^{t-1}(P) \neq S^t(P) \Rightarrow S^{t-1}(P) \subset S^t(P)$. Let $P \in P^{**}$ be such that $\forall P' \in P^{**}, \#T(P') \geq \#T(P)$ and $\#T(P') = \#T(P) \Rightarrow \max\{t < \overline{m}, [\{i \in N, m_i = t\} = \emptyset \text{ and } S^{t-1}(P) \neq S^t(P)]\} \geq \max\{t < \overline{m}, [\{i \in N, m_i = t\} = \emptyset \text{ and } S^{t-1}(P') \neq S^t(P')]\}$. Let $T = \max\{t < \overline{m}, [\{i \in N, m_i = t\} = \emptyset \text{ and } S^{t-1}(P) \neq S^t(P)]\}$ be the latest period for which $[\{i \in N, m_i = T\} = \emptyset \text{ and } S^{T-1}(P) \neq S^T(P)]$ is satisfied. By assumption, $T < \overline{m}$.

Let us define $A = S^{T-1}(P) \setminus S^T(P) \neq \emptyset$. Let $P' = (p'^{t'}_{i'})_{i' \in N, t' \in \mathbb{N}} \in \mathcal{P}$ be defined as:

$$\forall i' \in N, \forall t' \in \mathbb{N}, p'^{t'}_{i'} = \begin{cases} w_{i'} & \text{, if } i' \in A \text{ and } t' = T-1 \\ \min_{i \in S^T(P)} w_i & \text{, if } i' \in S^T(P) \text{ and } t' = T \\ p^{t'}_{i'} & \text{, otherwise} \end{cases}.$$

It is straightforward to check that

$$\begin{gathered} \Pi(P) - \Pi(P') = \\ (\beta^T - \beta^{T-1}) \left(\sum_{i \in A} w_i \omega_i \right) \\ +\beta^T \left(w_{S^{T-1}(P)} - w_{S^T(P)} \right) \sum_{i \in S^{T-1}(P)} \omega_i \\ -\beta^T w_{S^T(P)} \sum_{i \in A} \omega_i. \end{gathered}$$

Moreover, $P \in P^a$ implies $\Pi(P) - \Pi(P') \geq 0$. Then,

$$\sum_{i \in A} w_i \omega_i \leq \frac{\beta}{1-\beta} \left[\left(w_{S^{T-1}(P)} - w_{S^T(P)} \right) \left(\sum_{i \in S^{T-1}(P)} \omega_i \right) - w_{S^T(P)} \left(\sum_{i \in A} \omega_i \right) \right]. \tag{3}$$

Now, let us define $S^+ = \{i \in N, m_i \leq T+1 \text{ and } w_i \geq w_{S^{T-1}(P)}\} \setminus S^{T-1}(P)$ and $A^+ = \{i \in N, m_i \leq T+1 \text{ and } w_i \geq w_{S^T(P)} \text{ and } w_i < w_{S^{T-1}(P)}\} \setminus A$. Notice that A^+ and S^+ can be empty. Let $P'' = (p''^{t'}_{i'})_{i' \in N, t' \in \mathbb{N}} \in \mathcal{P}$ be defined as:

$$\forall i' \in N, \forall t' \in \mathbb{N}, p''^{t'}_{i'} = \begin{cases} \overline{w} + \delta & \text{, if } i' \in A \text{ and } t' = T \\ w_{S^{T-1}(P)} & \text{, if } i' \in S^+ \cup S^{T-1}(P) \text{ and } t' = T+1 \\ w'_i & \text{, if } i' \in A^+ \cup A \text{ and } t' = T+1 \\ p^{t'}_{i'} & \text{, otherwise} \end{cases}.$$

It is straightforward to check that

$$\Pi(P) - \Pi(P'') =$$
$$(\beta^T - \beta^{T+1}) \left(\sum_{i \in A} w_i \omega_i \right)$$
$$+ \beta^{T+1} \left(w_{S^T(P)} - w_{S^{T-1}(P)} \right) \left(\sum_{i \in S^{T-1}(P)} \omega_i + \sum_{i \in S+} \omega_i \right)$$
$$+ \beta^{T+1} \sum_{i \in A^+} \omega_i \left(w_{S^T(P)} - w_i \right)$$
$$+ \beta^{T+1} w_{S^T(P)} \sum_{i \in A} \omega_i$$

Moreover, $P \in P^a$ implies $\Pi(P) - \Pi(P'') \geq 0$. Then,

$$\sum_{i \in A} w_i \omega_i \geq \frac{\beta}{1-\beta} \left[\left(w_{S^{T-1}(P)} - w_{S^T(P)} \right) \left(\sum_{i \in S^{T-1}(P)} \omega_i + \sum_{i \in S+} \omega_i \right) + \sum_{i \in A^+} \omega_i \left(w_i - w_{S^T(P)} \right) - w_{S^T(P)} \sum_{i \in A} \omega_i \right]. \tag{4}$$

Using Equations 3 and 4, we have

$$\sum_{i \in A} w_i \omega_i = \frac{\beta}{1-\beta} \left[\left(w_{S^{T-1}(P)} - w_{S^T(P)} \right) \left(\sum_{i \in S^{T-1}(P)} \omega_i \right) - w_{S^T(P)} \left(\sum_{i \in A} \omega_i \right) \right].$$

And then, $\Pi(P) = \Pi(P') = \Pi(P'')$. Since, by assumption $P \in P^a$, we have, $P'' \in P^a$. Moreover, it is straightforward to check that P'' satisfies $\forall i \in N, \forall t \in \mathbb{N}$,

1. $i \notin S^t(P'') \Rightarrow {p''}_i^t = \overline{w} + \delta$,

2. $t = 0 \Rightarrow [i \in S^t(P'') \Rightarrow {p''}_i^t = w_i]$,

3. $t > 0 \Rightarrow [i \in S^t(P'') \Rightarrow {p''}_i^t = \min(w_i, \min_{t'<t, i' \in S^{t'}(P'')} w_{i'})]$.

Hence, $P'' \in P^{**} \subseteq P^*$. Moreover, $\#T(P') < \#T(P)$ or $[\#T(P'') = \#T(P)$ and $\max\{t < \overline{m}, [\{i \in N, m_i = t\} = \emptyset$ and $S^{t-1}(P) \neq S^t(P)]\} < \max\{t < \overline{m}, [\{i \in N, m_i = t\} = \emptyset$ and $S^{t-1}(P'') \neq S^t(P'')]\}$ contradicting the definition of P. □

Proof of Proposition 3: Follows directly from Lemma 11. □

Proof of Proposition 4: By Proposition 3, $\exists P \in P^a, \forall t > 1, S^t(P) = S^{t-1}(P)$. Obviously, by Lemma 7, $S^0(P) = \overline{S(P)} \in \mathcal{S}^0$. Let $S' \in \mathcal{S}_0$. Let $P' = ({p'}_{i'}^{t'})_{i' \in N, t' \in \mathbb{N}} \in \mathcal{P}$ be defined as:

$$\forall i' \in N, \forall t' \in \mathbb{N}, {p'}_{i'}^{t'} = \begin{cases} w_{i'} & \text{, if } i' \in S' \text{ and } t' = 0 \\ w_{S'} & \text{, if } i' \in S' \text{ and } t' > 0 \\ \overline{m} + \delta & \text{, otherwise} \end{cases}.$$

It is straightforward to check that

$$\Pi(P') = \sum_{i \in S'} w_i \omega_i + \frac{\beta}{1-\beta} w_{S'} \left(\sum_{i \in S'} \omega_i \right).$$

Since, $S' \in \mathcal{S}^0$ and $S \in \mathcal{S}^0$, $\Pi(P) = \Pi(P')$. Hence, $P' \in P^a$. Moreover, obviously, $\overline{S(P')} = S'$. □

Proof of Proposition 5: Let $P \in P^a$ be such that $\overline{S(P)} \notin \mathcal{S}^0$. By Lemma 6, $\tilde{P} \in P^a$ and by Lemma 5, $\overline{S(\tilde{P})} \notin \mathcal{S}^0$. By Lemma 10, $\exists P' \in P^*, \forall t > 0, S^{t-1}(P') = S^t(P') = \overline{S(\tilde{P})}$.

Then, by definition of P^*,

$$\Pi(P') = \sum_{i \in \overline{S(\tilde{P})}} w_i \omega_i + \frac{\beta}{1-\beta} w_{\overline{S(\tilde{P})}} \left(\sum_{i \in \overline{S(\tilde{P})}} \omega_i \right).$$

With Proposition 4, $P' \in P^* \subseteq P^a$ and $\overline{S(\tilde{P})} \notin \mathcal{S}^0$ is a contradiction. $\square$

Proof of Proposition 6: Let $P \in P^a$ and let $T \in \mathbb{N}$ be the earliest period for which $S^T(P) = \overline{S(P)}$ and $T > \overline{m}$. By Proposition 2, $\forall S \in \mathcal{S}^0, \overline{S(P)} \subset S$ or $S \subseteq \overline{S(P)}$. Assume $\forall S \in \mathcal{S}^0, \overline{S(P)} \subset S$. Let $S \in \mathcal{S}^0$.

By Lemmas 6 and 7, $\tilde{P} \in P^a$, T is the earliest period for which $S^T(\tilde{P}) = \overline{S(\tilde{P})}$ and $T > \overline{m}$. Moreover, $\overline{S(\tilde{P})} \subset S$. Let $S^+ = S \setminus \overline{S(\tilde{P})} \neq \emptyset$.

Let $P' = (p'^{t'}_{i'})_{i' \in N, t' \in \mathbb{N}} \in \mathcal{P}$ be defined as:

$$\forall i' \in N, \forall t' \in \mathbb{N}, p'^{t'}_{i'} = \begin{cases} w_i & \text{, if } i' \in S^+ \text{ and } t' = T+1 \\ w_S & \text{, if } i' \in S \text{ and } t' > T+1 \\ \tilde{p}^{t'}_{i'} & \text{, otherwise} \end{cases}.$$

It is straightforward to check that

$$\Pi(\tilde{P}) - \Pi(P') =$$

$$= \beta^{T+1} \left[\left(\sum_{i \in \overline{S(\tilde{P})}} w_i \omega_i \right) + \frac{\beta}{1-\beta} \left(w_{\overline{S(\tilde{P})}} \sum_{i \in \overline{S(\tilde{P})}} \omega_i \right) - \left(\sum_{i \in S} w_i \omega_i \right) - \frac{\beta}{1-\beta} \left(w_S \sum_{i \in S} \omega_i \right) \right]$$

By definition of $\mathcal{S}^0$ and since by assumption, $\overline{S(\tilde{P})} \notin \mathcal{S}^0$,

$$\left(\sum_{i \in S} w_i \omega_i \right) + \frac{\beta}{1-\beta} \left(w_S \sum_{i \in S} \omega_i \right) > \left(\sum_{i \in \overline{S(\tilde{P})}} w_i \omega_i \right) + \frac{\beta}{1-\beta} \left(w_{\overline{S(\tilde{P})}} \sum_{i \in \overline{S(\tilde{P})}} \omega_i \right).$$

Hence, $Pi(\tilde{P}) < Pi(P')$ contradicting the assumption that $\tilde{P} \in P^a$. $\square$

Definition 7

Let the function π be defined as follows: $\forall A \subseteq N$,

$$\pi(A) = \sum_{i \in A} \omega_i w_i + \frac{\beta}{1-\beta} \sum_{i \in A} \omega_i w_A.$$

Proof of Proposition 7: Let $P = (p_i^t)_{i \in N, t \in \mathbb{N}} \in P^a$ be such that $\overline{S(P)} \notin \mathcal{S}^0$. By Lemma 10, we can consider $\forall t > 1, S^{t-1} = S^t(P)$. By Lemma 5 and 6, we can consider $P \in P^*$. Let us define $A = S^0(P)$, $A^+ = \{i \in N, w_i \geq w_A\}$ and $B = S^1(P) \setminus A^+$. Then, by definition, $\overline{S(P)} = A^+ \cup B$. It is straightforward to check that

$$\Pi(P) = \sum_{i \in A} \omega_i w_i + \beta \sum_{i \in A^+} \omega_i (w_{A^+} - w_i) + \beta \pi(A^+ \cup B).$$

Let $S \in \mathcal{S}^0$.

I. Assume $A^+ \subseteq S$.

Let $P' = (p'^{t'}_{i'})_{i' \in N, t' \in \mathbb{N}} \in \mathcal{P}$ be defined as:

$$\forall i' \in N, \forall t' \in \mathbb{N}, p'^{t'}_{i'} = \begin{cases} w_i & \text{, if } i' \in S \setminus A^+ \text{ and } t' = 1 \\ w_S & \text{, if } i' \in S \text{ and } t' > 1 \\ p^{t'}_{i'} & \text{, otherwise} \end{cases}.$$

It is straightforward to check that

$$\Pi(P') = \sum_{i \in A} \omega_i w_i + \beta \sum_{i \in A^+} \omega_i (w_A - w_i) + \beta \pi(S).$$

Then, $\Pi(P) - \Pi(p') = \pi(A^+ \cup B) - \pi(S)$. However, $S \in \mathcal{S}^0$ and $\overline{S(P)} = A^+ \cup B \notin \mathcal{S}^0$ imply $\Pi(P) - \Pi(p') = \pi(A^+ \cup B) - \pi(S) < 0$. This contradicts the fact that $P \in P^a$.

II. Assume $S \subset A^+$.

Let us define $S_0 = \{i \in N, m_i = 0\}$, $S_1 = \{i \in S, w_i \geq w_{S_0}\}$ and $S_2 = S \setminus S_1$.

Let $P'' = (p''^{t'}_{i'})_{i' \in N, t' \in \mathbb{N}} \in \mathcal{P}$ be defined as:

$$\forall i' \in N, \forall t' \in \mathbb{N}, p''^{t'}_{i'} = \begin{cases} w_i & \text{, if } i' \in S_0 \text{ and } t' = 0 \\ w_{S_0} & \text{, if } i' \in S_1 \text{ and } t' = 1 \\ w_i & \text{, if } i' \in S_2 \text{ and } t' = 1 \\ w_S & \text{, if } i' \in S \text{ and } t' > 1 \\ \overline{w} + \delta & \text{, otherwise} \end{cases}.$$

It is straightforward to check that

$$\Pi(P'') = \sum_{i \in S_0} \omega_i w_i + \beta \sum_{i \in S_1} \omega_i (w_{S_0} - w_i) + \beta \pi(S).$$

Further,

$$\Pi(P) - \Pi(P'') =$$

$$\sum_{S \setminus S_0} w_i \omega_i - \sum_{A^+ \setminus A} w_i \omega_i$$

$$-\beta \left(\sum_{i \in S_1} \omega_i (w_{S_0} - w_S) + \sum_{i \in S_2} \omega_i (w_i - w_S) \right)$$

$$+\beta(\pi(A^+ \cup B) - \pi(S)) + (1 - \beta)(\pi(A^+) - \pi(S)).$$

$S \in \mathcal{S}^0$ and $\overline{S(P)} = A^+ \cup B \notin \mathcal{S}^0$ imply $\pi(A^+ \cup B) - \pi(S) < 0$ $S \in \mathcal{S}^0$ implies $\pi(A^+) - \pi(S) \leq 0$. By the definitions $\sum_{i \in S_1} \omega_i (w_{S_0} - w_S) + \sum_{i \in S_2} \omega_i (w_i - w_S) \geq 0$. Finally, by definition, $S \subset A^+$ implies $S \setminus S^0 = \{i \in S, m_i = 1\} \subseteq A^+ \setminus A = \{i \in A^+, m_i = 1\}$. Hence, $\sum_{S \setminus S_0} w_i \omega_i - \sum_{A^+ \setminus A} w_i \omega_i \leq 0$. Then, $\Pi(P) - \Pi(P'') < 0$ which contradicts the fact that $P \in P^a$. □

Proof of Proposition 8: The proof is similar to the proof of Proposition 7 with weak inequalities. □