A note on prudence and prevention against a health risk in a two-period model

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Abstract
We extend to the case of health risk the results obtained by Menegatti (2009) for prevention against a financial risk in a two-period model. When they face a pure health risk, involving no additional monetary risk, prudent individuals in terms of health make more effort of prevention than non-prudent ones. This result still holds when health deterioration also involve monetary costs provided that the individual is prudent and cross-prudent in both dimensions, health and wealth, or provided that he is multiprudent in the sense of Jouini, Napp and Nocetti (2013) and multivariate risk averse.

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1. Introduction

Health problems often have a double dimension. Not only do they directly reduce people’s welfare, but they can also lead to out-of-pocket health expenses. One way to limit health risks consists of adopting preventive behavior by making an effort to reduce the probability of their occurrence: sport, healthy lifestyle, vaccination, frequent medical check-ups. Many factors influence this preventive behavior, among which are the fear of illness, the subjective perception of its probability of occurrence or risk attitude. In this work, we are concerned with the influence of prudence, in the sense given by risk theory. The objective is to specify how prudence affects the preventive effort of the individual when the effort made at one period modifies the probability of the occurrence of the risk at a future period.

Prudence is now a well-known concept. Following Eeckhoudt and Schlesinger (2006) and Eeckhoudt, Rey and Schlesinger (2007), an individual is said to be prudent in one dimension (wealth or health) if he prefers to face a zero-mean risk in this dimension when he is in a good state (wealthy or healthy) rather than in the opposite case (less wealthy or less healthy). He is said to be cross-prudent in terms of wealth if he prefers to face a zero-mean health risk when he is richer rather than poorer, and cross-prudent in terms of health if he prefers to face a zero-mean risk on her
wealth when he is in good health rather than bad. If individual preferences can be described by a bivariate Von Neuman Morgensten utility function \( u(w, h) \) with wealth \((w)\) and health \((h)\) as arguments, the previous authors showed that an individual is prudent in terms of wealth if \( \frac{\partial^3 u}{\partial w^3} = u_{111} > 0 \), prudent in terms of health if \( \frac{\partial^3 u}{\partial h^3} = u_{222} > 0 \), cross-prudent with respect to health if \( \frac{\partial^3 u}{\partial w^2 \partial h} = u_{112} > 0 \) and with respect to wealth if \( \frac{\partial^3 u}{\partial w \partial h^2} = u_{222} > 0 \). Several works have shown that prudence in one of these forms matters for describing people’s choices facing health risks. It may affect their health investment (Dardanoni and Wagstaff, 1991), their precautionary savings to finance future health expenses (Kotlikoff, 1986; Palumbo, 1999; Kong et al., 2008; De Nardi et al., 2010), or their willingness to accumulate wealth to mitigate the harm caused by a zero-mean risk on their future health (Eeckhoudt et al., 2007; Macé, 2012). Bui et al., (2007) also showed that prudence, besides risk aversion, should lead society to allocate more health care resources to patients for which the expected outcomes after treatment are below average (see also Courbage (2010) on this point).

Recently, Jouini, Napp and Nocetti (2013) have extended the concept of prudence to the multivariate case. Consider an individual who derives utility from his income \((x_0)\) but also from \(n\) other attributes like for instance health status or the quality of environment. His utility function is well-behaved and given by \( v(x_0, x_1, ..., x_n) = v(x) \). He faces a multivariate risk \( \vec{\varepsilon} = (\vec{\varepsilon}_1, \vec{\varepsilon}_2, ..., \vec{\varepsilon}_n) \) with \( E[\vec{\varepsilon}] = 0 \) and he can invest a certain amount of money \( k > 0 \) in the \( n+1 \) attributes of the utility function according to the \((n+1)\)-dimensional rate of return \( \rho = (\rho_0, ..., \rho_n) \) with \( \rho_i \geq 0 \). Then, this individual is multivariate prudent in the direction of \( \rho \) if he prefers the lottery \( [x - k \rho \cdot x + \vec{\varepsilon}] \) to the lottery \( [x; x - k \rho + \vec{\varepsilon}] \) where in each lottery, the outcomes have equal probability. Put it simply, an individual is multivariate prudent if he prefers to face a multivariate risk \( \vec{\varepsilon} \) in the states of nature in which the value of the attributes is relatively high rather than low. The authors then showed that in an expected utility framework, multivariate prudence is equivalent to the convexity of \( \sum_{i=0}^{n} \rho_i \frac{\partial v}{\partial x_i} \) and so to the convexity of the different marginal utility functions \( v_0, v_1, ..., v_n \).

\[1\] As a consequence also, in the spirit of the initial definition of prudence given by Kimball (1990), the multivariate prudent individual who faces a multivariate risk \( \vec{\varepsilon} = (\vec{\varepsilon}_1, \vec{\varepsilon}_2, ..., \vec{\varepsilon}_n) \) with \( E[\vec{\varepsilon}] = 0 \) at the second period will save
Our results adds to the literature examining the links between prudence and prevention by making use of the concept of multivariate prudence. For financial risks, in a one-period framework in which efforts and their effect on probability are contemporaneous, a well-known result is that efforts of prevention may be reduced by a higher degree of prudence (Eeckhoudt and Gollier, 2005 and Courbage and Rey, 2006 for an application to health risk when people differ in their fear of sickness). In a two-period framework, when preventive actions precede the effect on the probability of loss, Menegatti (2009) showed however that a prudent individual does exert more effort of prevention because the effort no longer reduces the amount of wealth in the period in which the individual faces the risk. This two-period framework has been used since then in a few papers (Courbage and Rey, 2012; Eeckhoudt, Huang and Tzeng, 2012) and new results have been obtained about the link between prevention, risk aversion and prudence by Menegatti (2012) for financial risks and and substitutability between health prevention and cure by Menegatti (2013). A two-period framework is particularly relevant for health prevention in which the effort in prevention often precedes its effect on probability. A better health style, more frequent medical supervision, smoking cessation, etc.. typically exert their effect in the long term, after the efforts have been made. However, the link between prudence and prevention against health risks has not yet been explicitly explored in a two period-model. This paper fills the gap. We consider two cases i) a pure health risk (without additional financial cost) and ii) a risk of a simultaneous health and financial loss. In the first case, we show that an individual who is prudent in health makes more effort than a non-prudent one, contrary to what happens in a one-period framework. In the second case in which the individual faces a simultaneous health and financial loss, an individual makes more effort if he is prudent and cross prudent in the two dimensions, health and wealth also makes more effort. He makes also more effort if he is “only” multivariate prudent provided that he is also multivariate risk-averse as it was defined by Richard (1975): \( \frac{\partial^2 u}{\partial w \partial h} = u_{12} > 0 \). Our results thus extend to a bivariate health-wealth utility function the conclusions of Menegatti (2009) about the positive link between prudence and prevention.

The rest of the paper is organized as follows. Part 2 examines the consequences of prudence on optimal prevention against a pure health risk. The next part is devoted to the analysis of the optimal prevention in presence of a simultaneous health and financial loss risk. Part 4 discusses the results and concludes. The proofs are gathered at the appendix.

more at the first period to increase the value of the second period-attributes through the \((n + 1)\)-dimensional rate of return \( \rho = (\rho_0, \ldots, \rho_n) \).
2. Prudence and prevention against a pure health risk

Consider the following problem:

\[
Max \ V(e) = u(w_1 - e, h) + p(e)u(w_2, h - l) + [1 - p(e)]u(w_2, h)
\]  

(1)

where \( u(.) \) is the intra-period Von Neuman Morgenstern utility function, identical for both period to simplify, at least three times differentiable, with usual properties of decreasing marginal utilities with respect to both arguments and strictly concave \( u_iu_{zz} - u_{zz} > 0 \). \( w_1 \) and \( w_2 \) stand for initial level of wealth of both periods, \( h \) stand for health capital. It is measurable on a cardinal scale. \( p(.) \) is the probability of the health loss \( l \). In this pure health risk model, the individual does not endure additional financial cost. The probability \( p \) depends negatively on the level of efforts exerted by the individual at the first period to prevent it \( (p'(e) < 0) \). Efforts correspond to the purchase of health goods and medical goods or to the opportunity cost of the time used in activities like sports to the extent that they are used as instruments of prevention. To ensure that an internal solution exists, we assume furthermore as Menegatti (2009) and Eeckhoudt and Gollier (2005) did that \( V'(0) > 0 \) and \( V''(e) < 0 \). There is no saving. To simplify furthermore, the subjective discount rate is set to zero.

Our objective is to compare the level of effort chosen by an individual prudent with respect to health risk and an individual with similar preferences but neither prudent nor imprudent. First, denote \( e_n \) the level of risk chosen by the neutral-risk agent. If individuals differ only in terms of risk-aversion, then in the presence of risk, the risk-neutral agent chooses the same level of effort \( e_n \) than a risk-averse agent in the absence of risk. In the absence of risk, the loss \( l \) is sure and any effort \( e \) reduces this health loss by the proportion \( p(e) \). The utility function over the two periods becomes

\[
V(e) = u(w_1 - e, h) + u(w_2, h - p(e)l)
\]

which means that \( e_n \) satisfies:

\[
u_i(w_1 - e_n, h) = -p'(e_n)lu_2(w_2, h - p(e_n)l)
\]

(2)

To isolate the influence of prudence, we assume first as in Eeckhoudt and Gollier (2005) and Menegatti (2009) that the risk-neutral agent optimally selects a level of effort \( e_n \) such that \( p(e_n) = 1/2 \). We can then make use of the following lemma:
**Lemma 1**: Assume that the risk-neutral agent optimally selects a level of effort \( e_n \) such that \( p(e_n) = 1/2 \). Then, an individual with a utility function quadratic in health (and so neither prudent nor imprudent in this dimension) chooses the same level of effort (see Appendix A1 for the proof).

The derivative of \( V \) with respect to \( e_n \) is given by:

\[
V'(e_n) = -u_1(w_i - e_n, h) - p'(e_n)[u(w_2, h) - u(w_2, h-l)]
\]  

(3)

Replacing \( p'(e_n) \) by its expression given in equation (2) and assuming \( p(e_n) = 0.5 \), the condition can also be written:

\[
V'(e_n) = u_2 (w_i - e_n, h) \left[ \frac{u(w_2, h) - u(w_2, h-l)}{lu_2(w_2, h-0.5l)} - 1 \right]
\]  

(4)

It follows that \( V'(e_n) > 0 \) if \( u(w_2, h) - u(w_2, h-l) - lu_2(w_2, h-0.5l) > 0 \).

**Proposition 1**: Consider an individual facing a pure health risk at the second period and maximizing the program given by equation (1).

i) Suppose that the risk-neutral agent chooses a level of effort \( e_n \) such that \( p(e_n) = 1/2 \). Then the risk-averse individual chooses a level \( e' > e_n \) provided that he is prudent in health \( (u_{222} > 0) \).

ii) Suppose that the risk-neutral agent chooses a level of effort \( e_n \) such that \( p(e_n) \leq 1/2 \). Then the risk-averse \( (u_{22} < 0) \) and prudent \( (u_{222} > 0) \) agent chooses an effort larger than \( e_n \).

iii) Suppose that the risk-neutral agent chooses a level of effort \( e_n \) such that \( p(e_n) > 1/2 \). Then, the risk-averse \( (u_{22} < 0) \) and imprudent \( (u_{222} < 0) \) agent chooses an effort less than \( e_n \).

Therefore, prudence in health plays positively on health prevention as it does on health investment in a two-period model. As Menegatti (2009) and (2012) put it, the choice of the framework, a one or two-period model has radical consequences for the conclusions. We thus extend to the case of a
pure health risk in a bivariate utility function the result obtained by Menegatti (2009) for financial risk.

2. Health and monetary risk

Suppose now that in the case of health deterioration, the individual also endures a monetary cost $c$. This cost corresponds to out-of-pocket medical expenses or to a lesser productivity on the labor market. The new maximization program of the individual is given by:

$$\max \quad V(e) \equiv u\left(w_1 - e, h\right) + p(e)u\left(w_2 - c, h - l\right) + \left[1 - p(e)\right]u\left(w_2, h\right)$$  \hspace{1cm} (5)

There are now two risks, a risk of a monetary loss and a risk of health loss, which are perfectly correlated. Either both occur or none. We assume that the individual has the same preferences for these two risks. If he is risk-neutral (or risk-averse, or prudent) with one risk, then he is also risk-neutral (or risk-averse, or prudent) with the other. To examine the role of prudence, we follow the same steps as in the previous case of a pure health risk. The first step consists of observing that if individuals differ only in terms of risk-aversion to both risks, then the neutral-risk agent chooses the same level of effort $e_n$ than a risk-averse agent in the absence of risk. In the absence of risk, the utility function over the two periods becomes $V(e) = u(w_1 - e, h) + u(w_2 - p(e)c, h - p(e)l)$, which means that $e^*$ satisfies:

$$u_1(w_1 - e^*, h) = p^*(e_n)\left[lu_1(w_2 - p(e)\cdot c, h - p(e^*)l) + cu_1(w_2 - p(e^*)c, h - p(e^*)l)\right]$$  \hspace{1cm} (6)

As Menegatti (2009) noted, in a two-period model, the concavity of the utility of the risk-averse individual also reflects his desire to smooth consumption over time. And in the absence of health loss, the previous condition, according which in the absence of monetary risk, the level of prevention chosen by the risk neutral agent is also optimal for the risk averse agent, implies the same expected wealth for the two periods (see appendix A.3 on this point).

$$w_2 - p(e^*)c + e^* = w_1$$  \hspace{1cm} (7)

To isolate the role of prudence, assume again that the risk-neutral agent optimally selects a level of effort $e_n$ such that $p(e_n) = 1/2$. Again, it ensures that an individual with a utility function quadratic
in health and in wealth now (and so neither prudent nor imprudent with both risks) will choose the same level of effort (see lemma 2 appendix A.1).

The derivative of $V$ with respect to $e_n$ is now given by:

$$V'(e_n) = -u_i(w_i - e_n, h) - p'(e_n)[u(w_i, h) - u(w_i - c, h - l)]$$  \hspace{1cm} (8)

Replacing $p'(e_n)$ by its expression derived from equation (6), assuming $p(e_n) = 0.5$ and $w_i = w_2 - p(e_n)c + e_n$, the condition can also be written:

$$V'(e_n) = u_i\left( w_2 - 0.5c, h \right) \frac{u\left( w_2, h \right) - u\left( w_2 - c, h - l \right)}{lu_2(w_2 - 0.5c, h - 0.5l) + cu_1(w_2 - 0.5c, h - 0.5l)} - 1$$  \hspace{1cm} (9)

where $V'(e_n) > 0$ if $u\left( w_2, h \right) - u\left( w_2, h - l \right) - [lu_2(w_2 - 0.5c, h - 0.5l) + cu_1(w_2 - 0.5c, h - 0.5l)] > 0.$

Consider first the two following definitions applied to our well-behaved bivariate utility function $u(w, h)$.

**Definitions:**

- An individual is multivariate risk-averse if $u_{12} < 0$ (Richard, 1975)
- An individual is multivariate prudent if $u_1$ and $u_2$ convex (Jouini, Napp and Nocetti, 2013)

**Proposition 2:**

i) Suppose that $u_{12} = 0$. Then, if the individual is prudent in health and wealth $(u_{11} > 0, u_{22} > 0)$, he makes more effort of prevention than the risk-neutral individual.

ii) Suppose that $(u_{12} \neq 0)$. Then, if $u_1$ and $u_2$ convex, that is if the individual is multiprudent in terms of health and wealth in the sense of Jouini et al. (2013), the individual makes more effort of prevention than the risk-neutral individual.

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2 If we assume that $c=0$, we find again the previous condition. If instead, we assume $l=0$, we find the condition of Menegatti (2009).
iii) Suppose that the individual is prudent and cross-prudent in health and in wealth $(u_{11} > 0,u_{22} > 0,u_{12} > 0,u_{122} > 0)$, he makes more effort of prevent than the risk-neutral individual.

**Proposition 3:**

i) If $p(e_n) < 1/2$, a multivariate risk averse $(u_{12} < 0)$ agent who is multivariate prudent in the sense of Jouini et al. (2013) ($u_1$ and $u_2$ convex) or prudent and cross-prudent health and wealth selects an optimal level of effort greater than $e_n$.

ii) If $p(e_n) > 1/2$, a multivariate risk averse $(u_{12} < 0)$ agent who is multivariate imprudent in the sense of Jouini et al. (2013) ($u_1$ and $u_2$ concave) or imprudent and cross-imprudent makes less effort of prevention than the risk-neutral individual.

In the simplified case i) in which $u_{12} = 0$, the level of wealth does not affect the marginal utility of health, nor does a higher health may help to mitigate or amplify the harm caused by a zero-mean risk on health since third cross-derivatives are also nil. As a result, prudence in health and wealth are a sufficient condition to ensure that the individual will make more effort of prevention in a two-period model. When the utility function is health state dependent, that is when the marginal utility of wealth depends on health $(u_{12} \neq 0)$, the economic decision of the individual is now more complicated. When making this effort, he must take into account the fact that more of one attribute, for instance wealth, not only helps him to face a zero-mean risk in wealth at the second period if he is prudent in this dimension but also may amplify or mitigate the harm caused by a zero-mean risk on the other attribute, health in this example, depending on whether or not he is cross-prudent. Proposition 2.ii) indicates that the individual will make more effort if he the individual is multivariate prudent in terms of health and wealth and multivariate risk averse, and so even with a moderate level of cross-imprudence.

3. Conclusion

One of the best strategies for facing illnesses is to avoid them in the first place. Public authorities devote a lot of resources to health prevention. And it is important to understand how the risk attitude of individuals in general and prudence in particular affect their efforts of prevention. In these last decades, prudence has emerged as a relevant concept in the characterization of risk attitudes. Put it simply, prudence in general (including cross-prudence) reflects the preference of an
individual to face a zero-mean risk in one dimension (health or wealth) when he is a good state (wealthier or healthier) rather than in the bad one. Though prudence is not universal and though the empirical evidence is still scarce, prudence or cross-prudence seem the natural choice for many individuals. Crainich et al. (2013) showed for instance that even mixed risk-lovers are prudent as long as they are mixed-risk averse. The first work on the link between prudence and prevention against a financial risk made by Eeckhoudt and Gollier (2005) gave a somewhat surprising message: Prudence in terms of wealth leads people to invest less in prevention against wealth risk. Menegatti (2009) showed however that the conclusion is inverted in a two-period framework, when preventive actions precede the effect on the probability of loss. This paper extends this result to a simultaneous health and financial risk and shows that from a health prevention perspective, prudence in general turns out to be a desirable property. It also offers a new application of the concept of multiprudence developed by Jouini et al. (2013).

Appendix

A.1 Proof of lemma 1 and lemma 2.

Lemma 1: Assume that the risk-neutral agent optimally selects a level of effort $e_n$ such that $p(e_n)=1/2$. Then, an individual with a utility function quadratic in health (and so neither prudent nor imprudent in this dimension) chooses the same level of effort.

Lemma 2: Suppose that the agent risk-neutral with respect to health and wealth risk optimally selects a level of effort $e_n$ such that $p(e_n)=1/2$. Then an individual with a utility function quadratic in health and in wealth (and so neither prudent nor imprudent for each risk) chooses the same level of effort.

Let us start by Lemma 2. If individuals differ only with respect with risk-aversion, then in the presence of risk, the risk-neutral agent chooses the same level of effort than a risk-averse agent in the absence of risk, so that $e_n$ satisfies equation (2) in the main text. Assuming furthermore that $p(e_n)=0.5$, it implies that the risk-neutral individual selects $e_n$ such that:

$$u_1(w_1 - 0.5c, h) = -p(e_n)[Iu_2(w_2 - 0.5c, h - 0.5f) + cu_1(w_2 - 0.5c, h - 0.5f)]$$  \[A.1\]
where $u(\cdot)$ stands for the utility function of a non-neutral risk individual. Consider then a utility function quadratic in health and in wealth. The first-order condition of his maximization program indicates that he will choose $e_q$ such that

$$u_n(w_2 - 0.5c, h) = -p'(e_q)\left[u(w_2, h) - u(w_2 - c, h - l)\right] \tag{A.2}$$

Comparing conditions A.1 and A.2 shows that $e_q = e_n$ if

$$u(w_2, h) - u(w_2 - c, h - l) = lu_2(w_2 - 0.5c, h - 0.5l) + cu_1(w_2 - 0.5c, h - 0.5l).$$

To prove this equality, we can compute the Taylor expansions series of $u(w_2, h)$ at the second order and of $lu_2(\cdot)$ and of $cu_1(\cdot)$ at the first order. They give exact approximations since third partial derivatives and beyond with respect to each argument equal zero.

$$u(w_2 - c, h - l) = u(w_2, h) - lu_2(w, h) - cu_1(w, h) + l^2 \frac{u_{22}(w, h)}{2} + c^2 \frac{u_{11}(w, h)}{2} + u_{12}(w, h)c_l \tag{A3a}$$

$$u_2(w - 0.5c, h - 0.5l) = u_2(w, h) - 0.5lu_{22}(w, h) - 0.5cu_{12}(w, h) \tag{A3b}$$

$$u_1(w - 0.5c, h - 0.5l) = u_1(w, h) - 0.5cu_{11}(w, h) - 0.5lu_{12}(w, h) \tag{A3c}$$

It follows that $u(w_2, h) - u(w_2 - c, h - l) = lu_2(w_2 - 0.5c, h - 0.5l) + cu_1(w_2 - 0.5c, h - 0.5l)$ and so $e_q = e_n$.

Lemma 2 can be restricted to the case of a single risk (health or wealth). In the case of a pure health risk ($c = 0$) in particular we get the proof for lemma 1.

A2. Proof of proposition 1:

The derivative of the utility function of the risk-averse agent with respect to $e$, for $e = e_n$ and with $p(e_n) = 1/2$ is given by equation 4 in the main text:

$$V'(e_n) = u_1(w_1 - e_n, h)\left[\frac{u(w_2, h) - u(w_2, h - l)}{lu_2(w_2, h - 0.5l)} - 1\right] \tag{A.4}$$
The sign of $V'(e_n)$ is thus equal to the sign of $u(w_z, h) - u(w_z, h - l) - lu_z(w_z, h - 0.5l)$, which could also be written:\(^3\)

$$\text{sign}[V'(e_n)] = \text{sign} \left[ \int_{h-l}^{h} u_h(w, x)dx - lu_h(w, h - 0.5l) \right] \quad (A.5)$$

Denote to simplify $u_h = u_h(w, h - 0.5l)$ and $u_{hh} = u_{hh}(w, h - 0.5l)$. By definition, the tangent line to $u_h(w, h)$ at the point $(w, h - 0.5l, u_h(w, h - 0.5l))$ is given by:

$$T(w, x) = u_h + u_{hh} \cdot \left(x - [h - 0.5l]\right) = u_h + u_{hh}x - hu_{hh} + 0.5lu_{hh} \quad (A.6)$$

The integral of $T(w, x)$ over $[h-l, h]$ is given by

$$\int_{h-l}^{h} T(w, x)dx = u_h + u_{hh} \cdot \frac{h^2}{2} - u_{hh} \cdot \frac{(h-l)^2}{2} - hu_{hh}l + 0.5lu_{hh}l, \quad \text{or after simplification by:}$$

$$\int_{h-l}^{h} T(w, x)dx = u_h l \quad (A.7)$$

Replacing the expression of $u_h l$ given by A.4 in equation A.2 implies:

$$\text{sign}[V'(e_n)] = \text{sign} \left[ \int_{h-l}^{h} \left[u_h(w, x) - T(w, x)\right]dx \right] \quad (A.8)$$

If $u_h(w, x)$ convex with respect to $h$, that if $u_{hhh} > 0$ then, $u_h(w, x)$ lies above its tangent, $u_h(w, x) > T(w, x)$, which implies $V'(e_n) > 0$ : Proof of proposition I(i).

- Assume now that $p(e_n) \leq 1/2$. We want to find the sign of the following expression:

$$V'(e_n) = \left[ u_z \left( w_z - p(e_n) \right) c, h \right] - \left[ u_z \left( w_z, h - p(e_n) \right) l \right] - 1 \left[ u_2(w_z, h) - u_2(w_z, h - l) - lu_z(w_z, h - 0.5l) \right] \quad (A.9)$$

The sign of $V'(e_n)$ is that of $\left[ u_2(w_z, h) - u_2(w_z, h - l) - lu_z(w_z, h - 0.5l) \right]$. This last expression is decreasing with respect to the probability $p(e_n)$ if and only if the agent is risk-averse with respect

\(^3\) We adapt to the case of pure health risk the proof given by Menegatti (2009)
to health \((u_{22} < 0)\). So in that case, and with \(p(e_n) \leq 1/2\),
\[
\left[ u(w_2, h) - u(w_2, h - l) - lu_2(w_2, h - p(e_n)l) \right] > 0
\]
and the level of effort is greater than \(e_n\):
Proposition I(ii).

Assume finally that \(p(e_n) > 1/2\). When the agent is imprudent and risk-averse with respect to
health, the sign of \(V'(e_n)\) is non-positive, which means the optimal level of effort is less than \(e_n\):
Proposition I(iii)

A.3 Condition of equality of expected wealth for both periods when \(c > 0\)
Denote \(n(w, h)\) the utility function of the individual risk neutral with each risk. By definition, it
satisfies \(n(w_2, h) - n(w_2 - c, h - l) = \ln_2 + cn_1 - lc \cdot n_{12}\). The F.O.C of the program of the risk-neutral
individual in the presence of risk is given by:
\[
-n_1(w_i - e_n, h) - p'(e_n) [\ln_2 + cn_1 + lc \cdot n_{12}] = 0
\]  
(A.10)
The F.O.C for the program of a risk-averse individual in the absence of risk is given by:
\[
u_i(w_i - e^*, h) - p'(e^*) lu_2(w_2, h - p(e^*)l) = 0
\]  
(A.11)

\(p'(e^*)\) and \(p'(e_n)\) can be derived straightforwardly from equations A.1 an A.2. Since \(e^* = e_n\),
\(p'(e^*) = p'(e_n)\) and so:
\[
\frac{n_1(w_i - e_n, h)}{\ln_2 + cn_1 - lc \cdot n_{12}} = \frac{u_i(w_i - e_n, h)}{lu_2(w_2 - p(e_n)c, h - p(e_n)l) + cu_i(w_2 - p(e_n)c, h - p(e_n)l)}
\]  
(A.12)
For \(l = 0\), this condition can be simplified. It becomes \(\frac{1}{c} = \frac{u_i(w_i - e_n, h)}{cu_i(w_2 - p(e_n)c, h)}\), which implies that
\(w_2 - p(e^*)c + e^* = w_i\), which is the case studied by Menegatti (2009).

A.4 : Proof of proposition 2
In the bivariate case, the derivative of the utility function of the risk-averse agent with respect to \( e \), for \( e = e_n \) and with \( p(e_n) = 1/2 \) is given by equation 9 in the main text. It is positive if \( u(w_2, h) - u(w_2 - 0.5c, h - 0.5) + cu_1(w_2 - 0.5c, h - 0.5) > 0 \). Denote

\[
g(c, l) = u(w_2, h) - u(w_2 - c, h - l) - cu_1(w_2 - 0.5c, h - 0.5) - lu_2(w_2 - 0.5c, h - 0.5) \quad (A.13)
\]

Its first and second partial derivatives are given by:

\[
g_c = u_1(w_2 - c, h - l) - u_1(w_2 - 0.5c, h - 0.5) + 0.5cu_{11}(w_2 - 0.5c, h - 0.5) + 0.5lu_{12}(w_2 - 0.5c, h - 0.5) \quad (A.14a)
\]

\[
g_l = u_2(w_2 - c, h - l) - u_2(w_2 - 0.5c, h - 0.5) + 0.5lu_{22}(w_2 - 0.5c, h - 0.5) + 0.5cu_{12}(w_2 - 0.5c, h - 0.5) \quad (A.14b)
\]

\[
g_{cc} = -u_{11}(w_2 - c, h - l) + 0.5u_{11}(w_2 - 0.5c, h - 0.5) + 0.5u_{22}(w_2 - 0.5c, h - 0.5)
- 0.5^2cu_{111}(w_2 - 0.5c, h - 0.5) - 0.5^2lu_{112}(w_2 - 0.5c, h - 0.5) \quad (A.14c)
\]

\[
g_{cl} = -u_{12}(w_2 - c, h - l) + 0.5u_{12}(w_2 - 0.5c, h - 0.5) + 0.5u_{22}(w_2 - 0.5c, h - 0.5)
- 0.5^2lu_{122}(w_2 - 0.5c, h - 0.5) \quad (A.14d)
\]

\[
g_{ll} = -u_{22}(w_2 - c, h - l) + 0.5u_{22}(w_2 - 0.5c, h - 0.5) + 0.5u_{22}(w_2 - 0.5c, h - 0.5)
- 0.5^2lu_{122}(w_2 - 0.5c, h - 0.5) \quad (A.14e)
\]

\((0, 0)\) is a critical point of \( g(c, l) \) since \( g_c(0, 0) = g_l(0, 0) \). Note also that \( g_{cc}(0, 0) = g_{ll}(0, 0) = g_{cl}(0, 0) \). As a consequence, at the point \((0, 0)\), we have \( g_{cc}g_{ll} - g_{cl}^2 \). To know whether \((0, 0)\) is a local maximum or minimum, we can compute a Taylor expansion of the third order of \( g(c, l) - g(0, 0) = g(c, l) \). Note first that:

\[
u(w_2, h) - u(w_2 - c, h - l) = cu_1 - \frac{c^2}{2} u_{11} + \frac{c^3}{6} u_{111} + lu_2 - \frac{l^2}{2} u_{22} + \frac{l^3}{6} u_{222} - clu_{12} + \frac{c^2}{2} u_{112} + \frac{cl^2}{2} u_{122} \quad (A.15a)
\]

and

\[
cu_1(w_2 - 0.5c, h - 0.5) + lu_2(w_2 - 0.5c, h - 0.5) = c \left[ u_1 - \frac{c}{2} u_{11} + \frac{c^2}{8} u_{111} - \frac{l}{2} u_{12} + \frac{l^2}{8} u_{122} + \frac{c^2}{4} u_{112} + \frac{cl}{4} u_{122} \right] + \left[ u_2 - \frac{l}{2} u_{22} + \frac{l^2}{8} u_{222} - \frac{c}{2} u_{21} + \frac{c^2}{8} u_{211} + \frac{cl}{4} u_{212} \right] \quad (A.15b)
\]
Thus, \( g(c, l) = u(w_2, h) - u(w_2 - c, h - l) - c u_i(w_2 - 0.5c, h - 0.5l) - l u_2(w_2 - 0.5c, h - 0.5l) \)

\[
= c u_i - \frac{4c^2}{8} u_{11} + \frac{4c^3}{24} u_{111} + l u_2 - \frac{4l^2}{8} u_{22} + \frac{4l^3}{24} u_{222} - c l u_{12} + \frac{c^2 l}{2} u_{112} + \frac{c l^2}{2} u_{222} \\
- \left( c u_i - \frac{4c^2}{8} u_{11} + \frac{3c^3}{24} u_{111} + \frac{c l^2}{4} u_{112} + \frac{c^2 l}{4} u_{112} \right) - \left( l u_2 - \frac{4l^2}{8} u_{22} + \frac{3l^3}{24} u_{222} + \frac{c^2 l}{8} u_{211} + \frac{c l^2}{4} u_{222} \right) + c l u_{222}
\]

And so after simplification

\[
g(c, l) = \frac{c^3 u_{111} + l^2 u_{222} + 3c^2 l u_{112} + 3c l^2 u_{222}}{24} \quad (A.16)
\]

It is useful to rearrange the numerator of the previous expression to get:

\[
g(c, l) = \frac{1}{24} \left( c^2 u_{111} + 2c l u_{112} + l^2 u_{222} \right) + l \left( \frac{l^2 u_{222} + c^2 l u_{112} + 2c l u_{222}}{>0 \ if \ u_2 \ convex} \right) \quad (A.17)
\]

For \( c \) and \( l \) positive, \( g(c, l) - g(0, 0) > 0 \) if the numerator of the previous expression is positive, which is true:

i) if \( u_{111} > 0, u_{222} > 0, u_{112} > 0, u_{122} > 0 \) that is if the individual is prudent and cross-prudent in both dimensions: proposition 3(i)

ii) if \( u_1 \) and \( u_2 \) convex, that is if the individual is multivariate prudent: proposition 3(ii).

In the pure health risk case, \( c = 0 \) and \( g(c, l) = \frac{l^3 u_{222}}{24} > 0 \). It is positive if the individual is prudent in health. In the case of a pure financial risk, \( l = 0 \) and \( g(c, l) = \frac{c^3 u_{111}}{24} > 0 \). It is positive if the individual is prudent in wealth, which is the result obtained by Menegatti (2009).

A.4 : Proof of proposition 3

Suppose that \( p(e_n) \leq 1/2 \). We want to find the sign of the following expression:

\[
V'(e_n) = \left[ \frac{u_2(w_2 - p(e_n)c, h)}{c u_i(w_2 - p(e_n)c, h - p(e_n)l) + l u_2(w_2 - p(e_n)c, h - p(e_n)l)} - 1 \right] \times B \quad (A.17)
\]
with \[ B = \left[ u(w_2, h) - u(w_2 - c, h - l) - cu_1(w_2 - p(e_n) c, h - p(e_n) l) - lu_2(w_2, h - p(e_n) l) \right] \]

The sign of \( V'(e_n) \) is that of \( B \). To determine it, denote first

\[ \phi(p) = \left[ u(w_2, h) - u(w_2 - c, h - l) - cu_1(w_2 - p(e_n) c, h - p(e_n) l) - lu_2(w_2, h - p(e_n) l) \right] \]  
(A.18)

so that \( B = \phi(p(e_n)) \).

Then \[ \phi'(p) = c^2u_{11}(w_2 - pc, h - pl) + 2clu_{12}(w_2 - pc, h - pl) + l^2u_{22}(w_2 - pc, h - pl) \]
and if \( u_{11} < 0, u_{22} < 0 \) and \( u_{12} < 0 \), then \( \phi'(p) > 0 \).

Now assume that \( p(e_n) < l/2 \) and consider an agent such that \( u_{11} < 0, u_{22} < 0 \) and \( u_{12} < 0 \). Then given the previous result, it follows that \( B = \phi(p(e_n)) > \phi(0.5) \). Therefore \( V'(e_n) > 0 \), meaning that the optimal level of effort is greater than \( e_n \). The result when \( p(e_n) > 0.5 \) is obtained with the same approach.

References


