Long-term health investment and saving decisions and the "sad old age myth"

Serge Macé* – Institut catholique de Lille – LEM CNRS UMR 8179

November 2008

Abstract: To a certain extent, people adapt to the adverse consequences of old age as well as to most other undesired events. However, there is evidence to suggest that young people typically underestimate their ability to adapt to it (Lacey, Smith and Ubel, 2006). This paper explores the economic implications for health investment and saving decisions of this misprediction considered as a projection bias in the spirit of Loewenstein, O'Donoghue and Rabin (2003) in four small simple two-period models. Under standard and general assumptions, we show that when they view their future health level as given, this bias leads individuals to *undersave* if the marginal utility of consumption increases with health and *oversave* in the opposite case. When health becomes endogenous, and when people must choose during their youth between present consumption, saving and health investment, their inability to predict adaptation lead them to *overinvest* in their health at the expense of their present and future consumption as long as the marginal utility of health does not depend significantly now of the consumption level.

Key words: Changing tastes, intertemporal choice, misprediction, health investment, happiness, aging
JEL Classification: I12 – D1 – D91

I. INTRODUCTION

No one escapes from the laws of biology. Ageing is inherently associated with a declining beauty, a reduction of physical and cognitive functions, and an increasing probability of serious illnesses, disabilities or loss of close relatives, not to mention nostalgia for the past. But ageing is not systematically associated with declining utility if we except the very last years. Among recent studies for instance, Blanchflower and Oswald (2007) showed that happiness of Americans and Europeans is U-shape through the life cycle with a minimum in midlife, after controlling for income and after ruling out the possibility of a pure cohort effect[1]. Controlling for more factors like marital status, children, work status in particular, and excluding simple cohort effect again, Yang Yang (2008) found an increasing path this time for Americans[2]. Psychological studies like Carstensen et alii (1999) also indicate that at old age, the frequency of feeling sad or angry declines and when negative emotions occur, they don't last as long or they are more passive (Ross and Mirowksi, 2008).

* Contact: serge.mace@icl-lille.fr tel : 00 33 3 20 13 41 01. Institut Catholique de Lille, 41 rue du Port, 59 016 Lille Cedex (France)

[1] Johannesson and Gerdtham (1997) got the same conclusions on Swedish data; See also Afsa et Marcus (2008) on French data

[2] As Easterlin (2006) noted, it's important to keep in mind that the previous results have been obtained by controlling one or more age-related variables like income or marital status so that one cannot infer from the previous studies that happiness automatically increases with age. If one looks at various sociological, psychological and economic studies who do not systematically introduce this kind of control variable, there is no clear evidence about the exact form of the relationship between age and happiness. For instance, Argyle (2001) concluded to an increased path of happiness with age, Costa et al (1987) to a constant path and Mroczek and Spiro (2005) to a mild *inverted* U-curve with a peak at 65, higher than Easterlin which placed it at 44. This lack of agreement comes from the different samples, countries, scales and methodologies used (happiness studies, life satisfaction studies, life domain approach, affect studies, cross-sectional or longitudinal studies) and from the difficulty to neutralize birth cohort effects, age effect or a selection bias due to the higher mortality of less happy people. This debate however does not affect our point which is that declining health brought about by ageing does not imply strong declining utility because people adapt psychologically to it (see after).

Different factors have been invoked to explain this "old age paradox", in particular the increase in financial satisfaction, the new leisure time available for retired people, or even the leaving home of children. There is also actually an emerging consensus in the literature around the idea that an important part of the explanation lies in hedonic adaptation. To a certain extent, people actually seem to adapt to old age as well as to most events good or bad that affect their life, probably because adaptation or related phenomena like habituation or resilience have been favoured in our genes by natural selection. Considering only bad events, it is a classical result of psychology now that people tend to adapt at least partially for example to handicap, illnesses, lower incomes, romantic break-up or the death of a close relative[3]. There are different mechanisms through which people achieve this adaptation. Concerning ageing in particular, it seems that older people, because of their shorter time horizon, reassess their priorities in favour of immediate satisfying activities rather than pursing long-term goals[4]. They are also inclined to focus their attention, sometimes unconsciously, to positive information[5] confirming the common-sense idea that *wisdom comes with age.*

The other remarkable fact is that young people tend to underestimate their future adaptation to old age as well as to most other events to which they adapt[6]. By comparing self-reported happiness of younger and older adults with their estimates of happiness at different ages, Lacey H. P. and alii (2006) showed for instance that despite that older adults display the highest happiness, younger adults believe that happiness decreases with age (from 30 to 70), though less when they think about themselves than when they think about others. Put another way, a majority of young people believe in what can be labelled the « sad old age myth »[7]. To reveal and explain this myth and other related aging false beliefs, psychologists have for a long time now defined standard scales, like the Palmore Scale[8] and organizations like the American National Council on the Aging produced during several years an annual report on the "Myths and Realities of Aging"[9]. We could also get a good clue of the strength of this over-pessimist view of aging among young people by looking at all the efforts made in geriatric medicine schools to correct it[10].

To sum up, evidence indicates that 1) human beings possess a general ability to adapt partially to old age and 2) tend to have problems to anticipate it completely. From an economic point of view, since this double cognitive process modifies intertemporal preferences of individuals through their life cycle, it should have potentially significant economic implications that have not been explored yet to our knowledge. The objective of this paper is to identify clearly in different small two-period models (young/old) the consequences for saving and long-term health investment of the difficulty for people to correctly predict the evolution of their preferences by considering it as a projection bias in the spirit of Loewenstein, O'Donoghue and Rabin (2003). In many respects, a two period-model is too simplistic to thoroughly explore the consequences of the previous cognitive processes. However, it's not a bad start given that we know only a few things empirically about the timing of adaptation and virtually nothing about the evolution of the ability of people to predict it. And people often spontaneously refer to the dichotomy (young/old) when they explain in casual conversations their long-term financial or health investment. Even in this simple framework however, the conclusions are

[3] See in particular Brickman et al (1978) and Gilbert et al (1998) for two classical studies, Oswald & Powdthavee (2008) for a recent one on disability, Lykken, D. & Tellegen, A. (1996) for the strong version of adaptation and Diener, Lucas & Collon (2006) for a revised and nuanced version of adaptation.

[4] This is in substance the main postulate of the so-called "theory of socio-emotional selectivity" developed by Carstensen et alii (1999).

[5] Charles, Mather and Carstensen (2003) and Fung & Carstensen (2003) established experimentally these facts.

[6] This cognitive process is extremely general once again. Gilbert et al (1998) showed for instance that individuals underestimate their capacity to adapt to an electoral defeat, the failure to get a tenure at university and even a romantic breakup. See also Eastwick, Finkel, Krishnamurti and Loewenstein (2007) on this last point. It also affects medical decisions (Ubel et alii (2005) et Lacey et alii (2006)).

[7] Laura L. Carstensen also call it "the misery myth".

[8] Palmore, E. B. (1998). *The facts on aging quiz.* New York: Springer

[9] National Council on the Aging. *American Perceptions of Aging in the 21st Century The NCOA's Continuing Study of the Myths and Realities of Aging.* Washington

[10] Knapp & Stubblefield (2000)

not straightforward at first. If we underestimate our future adaptation to a declining health, will we save more or less, invest more or less in our health, and how does the substitutability between health and consumption affects our choice?

Part II describes the formalism used to introduce simply both adaptation to old age and the difficulty for individuals to predict it correctly. Part III presents the consequences of this imperfect prediction on saving and health investment decisions. Under standard and general assumptions, we show that when they view their future health level as given, this bias leads individuals to *undersave* if the cross partial derivative of the utility function between health and consumption is positive and *oversave* otherwise. When health becomes endogenous and in the more general model where people must choose during their youth between present consumption, saving and health investment, people *overinvest* in their health at the expense of present and future consumption as long as the marginal utility of health does not depend significantly of the consumption level. When it does, positively or negatively, the misprediction of adaptation has an undetermined effect on health investment and saving. Uncertainty about the productivity of the health investment does not change the qualitative nature of the previous results.

II. PREFERENCES

II.1. Utility function

In this model, the individual's utility at period t (with t = 0,1), u_t, is given by:

$$u_t = u(c_t,h_t)$$

where c_t is the consumption at the period t and h_t the « health » capital of the individual for the same period. More exactly, h must be understood here as a « youth capital » defined as a synthetic indicator of cognitive and physical aptitudes, beauty and health that are sooner or later inversely related to the biological age. With this definition, and because the reasoning is made with a typical individual, whatever the efforts he makes to "stay young", that is to preserve his health/youth (both terms are considered synonymous here), he will end up with less health when old : $h_1 < h_0$. The utility function has standard properties: For c et h positive, $\partial u/\partial c > 0$, $\partial u/\partial h > 0$, $\partial^2 u/\partial c^2 < 0$ and $\partial^2 u/\partial h^2 < 0$. u_t is null when either c_t or h_t are null. Finally, the utility function is concave $u_{cc}u_{hh} > u_{ch}^2$. Since there is no theoretical reason to favour one or the other, no particular sign is assigned to the cross partial derivative between c and h which may then be positive, negative or null.

II.2. Adaptation

The basic starting idea is that if the individual adapts at least partially to a declining "health", this has two consequences:

i) He draws more utility from a given level of health after adaptation
ii) The marginal utility of health after adaptation is lower than what would be observed with stable preferences. Having more physical abilities, lower probability of being ill, looking prettier is always a good thing, but it's reasonable to assume that it's less important if people adapt.

There are two ways to model this double assumption. The first one is to modify the utility function by assuming that the utility function after adaptation, let's note it $u^{\alpha}(.)$, is such that for any c_1 and h_1 positive:

$$u^{\alpha}(c_1,h_1) > u(c_1, h_1) \quad \text{and} \quad \frac{\partial u^{\alpha}(c_1,h_1)}{\partial h_1} < \frac{\partial u(c_1, h_1)}{\partial h_1}$$

The other possibility is to suppose that the utility function is unchanged but because of adaptation, all happens as if the subjective level of health (or perceived age), noted h_1^{α}, does not decrease as fast as his objective level h_1 (biological age). In this case, we've got:

$$u(c_1, h_1^{\alpha}) > u(c_1, h_1) \qquad \text{and} \qquad \frac{\partial\, u(c_1, h_1^{\alpha})}{\partial h_1} < \frac{\partial\, u(c_1, h_1)}{\partial h_1} \qquad (1)$$

This other way to model adaptation[11] that we will adopt now fits well incidentally with the fact that the gap between subjective perceived age and biological age tend to increase as people get older (see for instance Demakakos, Hacker and Gjonça, 2006).

The extent to which because of adapation h_1^{α} is higher than h_1 depends on the degree of adaptation. In order to introduce it simply, we adopt the following simplified linear simplification:

$$h_1^{\alpha} = h_1 + \alpha\,(\bar{h}_0 - h_1)$$

$\bar{h}_0$ is the individual's exogenous level of health at period 0 (youth) and $\alpha \in [0,1]$ is the degree of adaptation. When $\alpha = 0$, no adaptation occurs and $h_1^{\alpha} = h_1$. In the extreme case of a perfect adaptation (α=1), the deterioration of health capital has no effect on the subjective perception of the individual so that $h_1^{\alpha} = \bar{h}_0$. In the intermediate normal case ($0 < \alpha < 1$), $h_1^{\alpha} > h_1$ and the double assumption about adaptation (1) is satisfied.

II.3. Quality of the prediction of adaptation

The second core assumption of the model is that young people underestimate their ability to adapt to old age. In the spirit of Loewenstein, O'Donoghue, Rabin [2003], young people make a *projection bias* in the sense that the prediction of their future preferences is biased by their present ones. As a consequence, the utility predicted in the first period for the second one and for a given level of health h_1 is lower than the one that will be observed. Contrary to the authors however, this bias is not formalized by a deformation of the utility function but, building upon the previous formalism, by assuming that the individual forecasts only a part $(1\text{-}m) \in [0,1]$ of his degree of adaptation α. The *predicted* health level for t=1, written $\tilde{h}_1$, is then given by:

$$\tilde{h}_1 = h_1 + \alpha_m\,(\bar{h}_0 - h_1) \qquad \text{where } \alpha_m = \alpha(1 - m) \qquad (2)$$

The parameter *m* measures the degree of *m*isprediction of adaptation. The higher it is, the closer the individual's predicted preferences from initial ones. The parameter α_m is the degree of *predicted adaptation*. It increases with α but decreases with m. As long as $0 < m < 1$, the predicted health capital, $\tilde{h}_1$, lies between the true health capital after adaptation, h_1^{α}, and the one that the individual would have had without adaptation if his future tastes were identical to his current ones, h_1:

$$h_1^{\alpha} > \tilde{h}_1 > h_1 \qquad \text{for } \alpha > 0 \text{ and } 0 < m < 1 \qquad (3)$$

In other words, the person predicts the direction of the change involved by adaptation ($\tilde{h}_1 > h_1$) but underestimates its intensity ($h_1^{\alpha} > \tilde{h}_1$). This assumption also implies that:

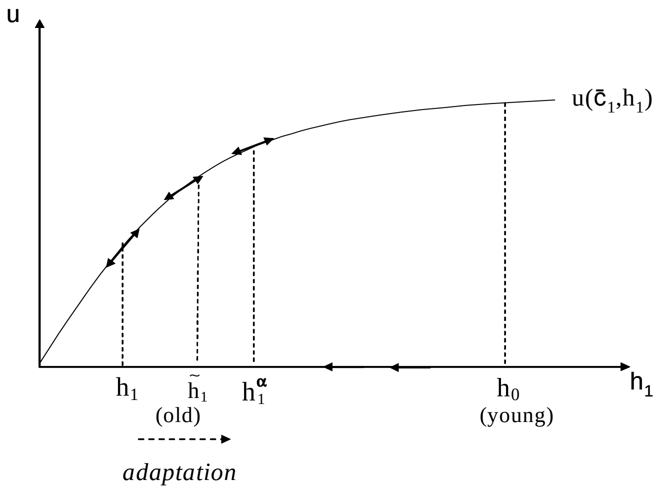
[11] Appendix A compares the two ways of formalizing adaptation using a linear specification

$$u(c_1, h_1^{\alpha}) > u(c_1, \tilde{h}_1) > u(c_1, h_1) \qquad \text{for any } c_1 > 0$$

and so $$\frac{\partial u(c_1,h_1^{\alpha})}{\partial h_1^{\alpha}} < \frac{\partial u(c_1, \tilde{h}_1)}{\partial h_1} < \frac{\partial u(c_1,h_1)}{\partial h_1}$$

The predicted marginal utility of health is lower than the one that would be attained without adaptation but higher that the one that will be eventually observed after adaptation. The following figure presents the typical intermediary case of a partial adaptation and of the partial prediction only of this adaptation: $0 < \alpha_m < \alpha < 1$

Figure 1 : Future absolute and marginal utilities when the individual underestimate their ability to adapt to old age



With the previous assumptions and after introducing a constant positive discount rate δ, the *predicted* total utility of the person over the two periods calculated at time 0 is then given by:

$$\tilde{U} = u(c_0, h_0) + \frac{1}{1+\delta} u(c_1, \tilde{h}_1) \qquad (4)$$

III. SAVING AND HEALH INVESTMENT DECISIONS IN TWO-PERIOD MODELS (YOUNG /OLD) WHEN PEOPLE MISPREDICT THEIR ADAPTATION

III.1 Saving decision without health investment

To get a first intuition about the effects on saving of adaptation and of the inability of people to predict it correctly, let us first assume a simple two-period model where the interest rate (r), the discount rate (δ), the incomes for both periods (y_0, y_1) are exogenous. The objective health levels for both periods are also exogenous with $\bar{h}_1 < \bar{h}_0$. This last hypothesis for h_1 will be abandoned afterwards. However, it may not be too unrealistic for those people who tend to have a fatalistic view by considering that they can't change many things about their future health level beyond a standard health investment.

Let's note $\tilde{\bar{h}}_1$ the exogenous predicted level of health capital after adaptation. Using (2), we've got $\tilde{\bar{h}}_1 = \bar{h}_1 + \alpha_m (\bar{h}_0 - \bar{h}_1)$. Under the previous assumptions, and given (4), the program of the individual follows as:

$$\text{Max}\,\Big|_{c_0,c_1} \quad \tilde{U}(c_0,c_1) = u(c_0, \bar{h}_0) + \frac{1}{1+\delta}\, u(c_1, \tilde{\bar{h}}_1)$$

$$\text{subject to} \quad c_1 = y_1 + (y_0 - c_0)(1+r) \qquad ; \; c_0 \geq 0 \;\text{ et } c_1 \geq 0$$

The three equilibrium first-order conditions, sufficient because of the concavity of $\tilde{U}$, of the Lagrangian associated with this program are given by[12]:

$$\frac{\partial u(c_0^e, \bar{h}_0)}{\partial c_0} - \mu^e \equiv 0$$

$$\frac{1}{1+\delta}\,\frac{\partial u(c_1^e, \tilde{\bar{h}}_1)}{\partial c_1} - \mu^e \equiv 0$$

$$y_1 + (y_0 - c_0^e)(1+r) - c_1^e \equiv 0$$

where μ^e is the Lagrangian multiplier.

To examine the consequences on the saving decision of the ability of individuals to adapt and of the difficulty to anticipate it correctly, we just have to express the equilibrium variables c_0^e, c_1^e et μ^e as functions of α_m, then derive the previous system of identities. We obtain[13]:

$$\underbrace{\begin{pmatrix} \tilde{U}_{11} & 0 & -1 \\ 0 & \tilde{U}_{22} & -1 \\ -(1+r) & -1 & 0 \end{pmatrix}}_{J} \begin{pmatrix} dc_0^e/d\alpha_m \\ dc_1^e/d\alpha_m \\ d\mu^e/d\alpha_m \end{pmatrix} = \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix}$$

where $\tilde{U}_{11}$ and $\tilde{U}_{22}$ are the second partial derivatives of $\tilde{U}$ respective to c_0 and c_1

and with $\quad a = -\dfrac{1}{1+\delta}\,\dfrac{\partial^2 u}{\partial \tilde{\bar{h}}_1 \partial c_1}\,(\bar{h}_0 - \bar{h}_1)$

Using Cramer's Rule, we get:

$$\frac{dc_0^e}{d\alpha_m} = \frac{a}{\det J} \qquad \text{and} \qquad \frac{dc_1^e}{d\alpha_m} = \frac{-a(1+r)}{\det J}$$

The determinant of J is positive[14]. Because $r > 0$, c_0 and c_1 vary in opposite ways for $a \neq 0$ and their direction depends on the sign of a. Since the sign of a is the opposite of the sign of the equilibrium cross partial derivative between c_1 and h_1 (written u_{ch} now for simplicity), we conclude that

[12] Because of the no satiety assumption with respect to c and h, we can neglect here the positivity constraint on c_0 and c_1

[13] Precisely after deriving the equilibrium variables with respect to α_m, we get

$$\tilde{U}_{11}\,\partial c_0^e/\partial\alpha_m - d\mu^e/d\alpha_m = 0 \qquad \text{with } \tilde{U}_{11} = \partial^2 u(c_0, \bar{h}_0)/\partial c_0^2 < 0$$

$$\tilde{U}_{22}\,\partial c_1^e/\partial\alpha_m - d\mu^e/d\alpha_m = a \qquad \text{with } \tilde{U}_{22} = (1/1+\delta)\,\partial u^2(c_1, \tilde{\bar{h}}_1)/\partial c_1^2 < 0 \;\text{ and } a = -(1/1+\delta)\,\partial^2 u/\partial \tilde{\bar{h}}_1 \partial c_1\,(\bar{h}_0 - \bar{h}_1)$$

$$-(1+r)\,\partial c_0^e/\partial\alpha_m - \partial c_1^e/\partial\alpha_m = 0$$

which can be written in matrix form as in the central text

[14] This matrix is not identical to the bordered Hessian matrix associated to the Lagrangian of the program since $\tilde{L}_{11} = \tilde{U}_{11}$ but $\tilde{L}_{22} = \tilde{U}_{22} - a$

$$\text{sign}\left(\frac{dc_1^e}{d\alpha_m}\right) = \text{sign } u_{ch} = - \text{sign}\left(\frac{dc_0^e}{d\alpha_m}\right)$$

To understand the meaning of this equivalence, suppose first that the marginal utility of consumption increases with health capital ($u_{ch} > 0$). Remembering that α_m decreases when m (which measures the degree of underestimation by the individual of his adaptation) increases, we get:

$$\frac{dc_1^e}{d\alpha_m} \geq 0 \quad \text{and} \quad \frac{dc_1^e}{dm} \leq 0 \qquad\qquad \text{for} \quad u_{ch} > 0$$

The future consumption level chosen by the individual will be all the higher that α is high but m is low. Actually, with adaptation, all happens as if the level of health at the second period was higher, which increases the utility of future consumption and so the incentive to save if $\partial^2 u/\partial\tilde{\tilde{h}}_1\partial c_1 > 0$. As long as they underestimate their degree of adaptation ($0 < m < 1$) however, individuals do not raise enough their saving. They *undersave*. Put it simply, because they perceive that old age, after all, will not be so bad, they put more money aside for their old days, but not as much as they would if they correctly perceived their adaptation. We can sum up by writing:

$$c_1^e\left(\begin{matrix}\textit{if no adapation}\\ \alpha_m=0\end{matrix}\right) < c_1^e\left(\begin{matrix}\textit{if mispredicted adapation}\\ 0 < \alpha_m < \alpha\end{matrix}\right) < c_1^e\left(\begin{matrix}\textit{if correctly mispredicted adapation}\\ \alpha_m = \alpha\end{matrix}\right)$$

Consequences are inverted if the marginal utility of consumption decreases with ageing. The saving choice remains optimal (if we compare it with the perfect prediction case) only if the marginal utility of consumption does not depend on health. Thus, the sign of the cross partial derivative plays a crucial role in this model and in the following ones. All things equal and independently of the discount factor, does the individual prefer to consume more when he is young and healthy or older and less healthy ? We will discuss briefly this point in the conclusion.

III.2. Health investment decision without saving

We now abandon the previous hypothesis of an exogenous future level of health by assuming that the youth/health capital deteriorates between the two periods at a rate which depends on the efforts to preserve it. In traditional health demand model like Grossman [1972], efforts include health goods and services but also other activities like sport and leisure which also affect our long-term health. In this model, we will refer only on the second type of variable: the time invested in one's health. By working less intensively and by simply increasing sports activities and leisure time, the individual ends up in better health when old. It's better however to interpret this assumption by stating that besides sports and leisure time, the individual buys in reality health goods (medicine, doctors services, cosmetic products or surgery, healthy foods, etc..) in the simplified case where we exclude any variation in their relative prices regarding to other consumption goods. To model it, we suppose here that at the first period, the individual must allocate his time between his health production in proportion θ_h and his labour supply for present consumption in proportion $\theta_c = 1 - \theta_h$. There is no saving[15]. Specifically, we suppose that the (objective) health production function follows as:

$$h_1 = h_1(\theta_h)$$

[15] We would get exactly the same qualitative results afterwards by considering that saving is just constant, positive or negative but not necessarily equal to 0. In that case the amount of time allocated to the labour market for present consumption would be equal to $1 - \theta_h - \bar{\theta}_s$ with $\bar{\theta}_s$ being the amount to time allocated to the labour market for to earn a revenue allocated to saving.

with $\partial h_1/\partial\theta_h > 0$; $\partial^2 h_1/\partial\theta_h^2 < 0$: These hypotheses guarantee the concavity of u and so $\tilde{U}$ with respect to θ_h and θ_c now. We assume that $h_1(0) = 0$ and $h_1(1) < \bar{h}_0$: Even if the individual allocates all his first-period time to preserve his health, he will end up with less of it.

The predicted health stock is now given by:

$$\tilde{h}_1 = h_1(\theta_h) + \alpha_m (\bar{h}_0 - h_1(\theta_h))$$

The time which is not invested in health is used on the labour market to earn an income which is spent for present consumption. We assume that the consumption level of the first period c_0 depends now positively (and linearly) on θ_c. We also put aside to start the possibility for the individual to transfer consumption through periods so that with $c_1 = \bar{c}_1$ (constant).

Under these assumptions, the program of the individual is now given by:

$$\text{Max}\,|_{\theta_c,\theta_h}\ \tilde{U}(\theta_c,\theta_h) = u(c_0(\theta_c), \bar{h}_0) + \frac{1}{1+\delta} u(\bar{c}_1, \tilde{h}_1(\theta_h))]$$

$$\text{subject to}\quad \theta_c + \theta_h = 1 \qquad \text{and} \qquad \theta_c \geq 0\ ;\ \theta_h \geq 0$$

The 3 first order conditions, sufficient again, of the Lagrangian are given by[16]:

$$\frac{\partial u(c_0, \bar{h}_0)}{\partial c_0}\frac{\partial c_0(\theta_c^e)}{\partial \theta_c} - \mu^e \equiv 0$$

$$\frac{1}{1+\delta}\frac{\partial u(\bar{c}_1, \tilde{h}_1)}{\partial \tilde{h}_1}\frac{\partial \tilde{h}_1}{\partial\theta_h} - \mu^e \equiv 0$$

$$1 - \theta_c^e - \theta_h^e \equiv 0$$

Given the strict concavity assumptions for $u(.)$ and $h_1(.)$ and the linearity of $c_0(\theta_c)$, the second-order condition is satisfied. The equalization of the two first equations implies that at the equilibrium, the individual must be indifferent between the allocation of one unit of time to the present consumption/production and one unit of time allocated to health production for the second period.

To examine the consequences of a mispredicted adaptation to lower health, we could follow again the procedure used in III.1 by expressing the equilibrium variables θ_c^e, θ_h^e et μ^e as functions of α_m then by deriving the system formed by the previous identities: We obtain again a 3 equations matrix system:

$$\underbrace{\begin{pmatrix} \tilde{U}_{11} & 0 & -1 \\ 0 & U_{22} & -1 \\ -1 & -1 & 0 \end{pmatrix}}_{J}\begin{pmatrix} d\theta_c^e/d\alpha_m \\ d\theta_h^e/d\alpha_m \\ d\mu^e/d\alpha_m \end{pmatrix} = \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix}$$

where $\tilde{U}_{11} < 0$ et $\tilde{U}_{22} < 0$ are now the second partial derivatives of $\tilde{U}(\theta_c,\theta_h)$[17] and

$$b^{18} = \frac{1}{1+\delta}\frac{\partial h_1}{\partial\theta_h}\left(\frac{\partial u(\bar{c}_1,\tilde{h}_1)}{\partial \tilde{h}_1} - (1-\alpha_m)\frac{\partial u^2(\bar{c}_1,\tilde{h}_1)}{\partial \tilde{h}_1^2}(\bar{h}_0 - h_1)\right) > 0$$

---

[16] Again also, the no satiety condition with respect to c and h allows us to neglect the positivity constraint on θ_c and θ_h

[17] $\tilde{U}_{11} = \frac{\partial^2 u(c_0, \bar{h}_0)}{\partial c_0^2}\left(\frac{\partial c_0}{\partial \theta_c}\right)^2 + \frac{\partial u(c_0, \bar{h}_0)}{\partial c_0}\frac{\partial^2 c_0(\theta_c^e)}{\partial \theta_c^2} < 0$ and $\tilde{U}_{22} = \frac{1}{1+\delta}\left(\frac{\partial u^2(\bar{c}_1\, \tilde{h}_1)}{\partial \tilde{h}_1^2}\left(\frac{\partial \tilde{h}_1}{\partial\theta_h}\right)^2 + \frac{\partial u(\bar{c}_1\, \tilde{h}_1)}{\partial \tilde{h}_1}\frac{\partial^2 \tilde{h}_1}{\partial\theta_h^2}\right) < 0$

[18] This value for b comes from the derivation of the second of the third order conditions with respect to α_m.

$$\frac{\partial}{\partial\alpha_m}\left(\frac{1}{1+\delta}\frac{\partial u(\bar{c}_1, \tilde{h}_1)}{\partial \tilde{h}_1}\frac{\partial \tilde{h}_1(\theta_h^e(\alpha_m))}{\partial\theta_h} - \mu^e(\alpha_m)\right) = 0$$

The determinant of J is positive. Using Cramer's rule, we've got:

$$\frac{d\theta_c^e}{d\alpha_m} = \frac{b}{\det J} \quad > 0 \tag{5a}$$

And $$\frac{d\theta_h^e}{d\alpha_m} = \frac{-b}{\det J} \quad < 0 \tag{5b}$$

Thus, as soon as the individual adapts ($\alpha > 0$), he no longer needs to allocate as much time to preserve his health : $d\theta_h^e/d\alpha_m < 0$ and $d\theta_c^e/d\alpha_m > 0$. Actually, adaptation reduces marginal utility of health for the second period for any level of health, which reduces the incentive to invest in it at the first period. However, as long as the individual underestimates this adaptation ($m > 0$, which reduces α_m), he *overinvests* in his health. It's worth noting that the previous conclusion is independent on the fact that the marginal utility of health depends positively or on the contrary negatively on the consumption level. The cross partial derivative does not play any role in (5a) and (5b) because in this model, the future consumption level being given, the individual cannot modify it in order to influence the marginal utility of health.

This rather surprising result may be stated simply: Why spending so much time and money in cosmetic creams and surgery, running under the rain, and sweating in sports club if at the end of the day, things are not that bad, thanks to adaptation. We should certainly do all those activities but their long-term effect on our utility is not necessarily as important at we may think if we underestimate adaptation. This conclusion holds as long of course that we're not victims of other cognitive biases like for instance time-inconsistent preference for immediate gratification which would lead us on the contrary to underinvest in our health.

Note : Introducing uncertainty on health investment productivity

In the previous model, the individual was inaccurately predicting the gap between his subjective and an objective and *certain* health stock. Given that in reality, health investment possess a stochastic component, it's interesting to ask whether this uncertainty changes the previous result and how. To answer, let's model this stochastic component of health investment in the simplest way by considering that the health capital of the second period, written now ψ_1, *is* a random variable defined by:

Remembering first that $\tilde{h}_1(\theta_h) = h_1(\theta_h) + \alpha_m(\bar{h}_0 - h_1(\theta_h))$

which implies : $\frac{\partial \tilde{h}_1}{\partial \alpha_m} = -h_1 + (1-\alpha_m)\frac{\partial h_1}{\partial \theta_h}\frac{\partial \theta_h}{\partial \alpha_m} + \bar{h}_0 = \frac{\partial \tilde{h}_1}{\partial \theta_h}\frac{\partial \theta_h}{\partial \alpha_m} + (\bar{h}_0 - h_1)$

and $\frac{\partial}{\partial \alpha_m}\left(\frac{\partial \tilde{h}_1}{\partial \theta_h}\right) = \frac{\partial}{\partial \alpha_m}\left((1-\alpha_m)\frac{\partial h_1}{\partial \theta}\right) = -\frac{\partial h_1}{\partial \theta_h} + (1-\alpha_m)\frac{\partial^2 h_1}{\partial \theta_h^2}.\frac{\partial \theta_h}{\partial \alpha_m}$

The derivation of the previous condition with respect to α_m let us write:

$$\frac{\partial}{\partial \alpha_m}\frac{1}{1+\delta}\left(\frac{\partial u(c_1, \tilde{h}_1)}{\partial \tilde{h}_1}\right)\frac{\partial \tilde{h}_1}{\partial \theta_h} + \frac{1}{1+\delta}\frac{\partial u}{\partial \tilde{h}_1}\frac{\partial}{\partial \alpha_m}\left(\frac{\partial \tilde{h}_1}{\partial \theta_h}\right) - \frac{d\mu^e}{d\alpha_m} = 0$$

$$\rightarrow \frac{1}{1+\delta}\frac{\partial^2 u}{\partial \tilde{h}_1^2}.\left(\frac{\partial \tilde{h}_1}{\partial \theta_h}\frac{\partial \theta_h^e}{\partial \alpha_m} + (\bar{h}_0 - h_1)\right)\frac{\partial \tilde{h}_1}{\partial \theta_h} + \frac{1}{1+\delta}\frac{\partial u}{\partial \tilde{h}_1}\left(-\frac{\partial h_1}{\partial \theta_h} + (1-\alpha_m)\frac{\partial^2 h_1}{\partial \theta_h^2}.\frac{\partial \theta_h^e}{\partial \alpha_m}\right) - \frac{d\mu^e}{d\alpha_m} = 0$$

$$\rightarrow \frac{1}{1+\delta}\left(\frac{\partial^2 u}{\partial \tilde{h}_1^2}\left(\frac{\partial \tilde{h}_1}{\partial \theta_h}\right)^2.\frac{\partial \theta_h^e}{\partial \alpha_m} + \frac{\partial u}{\partial \tilde{h}_1}\frac{\partial^2 \tilde{h}_1}{\partial \theta_h^2}\right).\frac{\partial \theta_h^e}{\partial \alpha_m} + \frac{1}{1+\delta}\frac{\partial^2 u}{\partial \tilde{h}_1^2}\frac{\partial \tilde{h}_1}{\partial \theta_h}(\bar{h}_0 - h_1) - \frac{1}{1+\delta}\frac{\partial u}{\partial \tilde{h}_1}\frac{\partial h_1}{\partial \theta} - \frac{d\mu^e}{d\alpha_m} = 0$$

$$\rightarrow \tilde{U}_{22}\frac{\partial \theta_h}{\partial \alpha_m} - \frac{d\mu^e}{d\alpha_m} = b \qquad \text{with } b = \frac{1}{1+\delta}\frac{\partial h_1}{\partial \theta_h}\left(\frac{\partial u}{\partial \tilde{h}_1} - (1-\alpha_m)\frac{\partial u^2}{\partial \tilde{h}_1^2}(\bar{h}_0 - h_1)\right)$$

$$\psi_1(\theta_\psi) = \begin{cases} h_1(\theta_\psi) - \varepsilon \text{ with a probability p} \\ h_1(\theta_\psi) + \varepsilon \text{ with a probability 1 - p} \end{cases}$$

with θ_ψ : proportion of the time available at the first period that the individual chooses to allocate to his health production in an uncertain context.

and $\varepsilon > 0$ and such that $h_1(1) + \varepsilon < \bar{h}_0$

No other hypotheses of the previous model are changed. We've got the same accumulation function $h_1(.)$ and we still have $c_1 = \bar{c}_1$ (given). The individual must maximize his predicted *expected* utility over the two periods:

$$\text{Max}_{|\theta_c,\theta_\psi} E\tilde{U}^0 = u(c_0(\theta_c), \bar{h}_0) + \frac{1}{1-\delta} E[u(\bar{c}_1, \tilde{\psi}_1(\theta_\psi))]$$

Subject to $\theta_c + \theta_\psi = 1$ and $\theta_c \geq 0$; $\theta_\psi \geq 0$

$$\text{with } E[u(\bar{c}_1, \tilde{\psi}_1(\theta_\psi))] = p\, u(\bar{c}_1, \tilde{h}_1(\theta_\psi) - \varepsilon) + (1-p)\, u(\bar{c}_1, \tilde{h}_1(\theta_\psi) + \varepsilon)]$$

The term "predicted expected utility" needs a comment. It simply says that the individual tries to predict at the first period his adaptation to a future lower endogenous health capital, whatever the uncertainty attached to this future health capital. $E[\partial u/\partial \psi_1]$ can be equal, lower or higher than $\partial u(c_1,\tilde{h}_1)/\partial \tilde{h}_1$ depending of the sign of the second derivative of that function, that is the sign of the third derivative of u, $\partial u^3(c_1, \tilde{h}_1)/\partial \tilde{h}_1^3$, sign which is directly linked to the type of risk aversion and to the existence or not of a prudent behaviour in the sense of Kimball (1990). If the individual is prudent (third derivative positive), he will makes a precautionary health investment in the same way he constitutes precautionary saving when facing uncertain future income. From our perspective, the important point however is that given that the optimal level of time that must be allocated to health production in a world of uncertainty, let's write it θ_ψ^* (calculated with m = 0), moves in the same direction as the chosen level (θ_ψ^e) depending on risk preferences, the individual still *overinvests* in his health as long as he underestimates his ability to adapt to old age. Thus, uncertainty about the productivity of the health investment does not change the qualitative nature of the previous results synthesised in equations (5a) and (5b).

III.3 Allocating between present consumption, health investment and saving

We now put aside uncertainty on health investment and complete the previous model by introducing financial markets, that is the possibility for the individual to use part of his wage earned at the first period to invest in financial assets at the exogenous interest rate r. We consider first that the amount of time allocated to the labour market for present consumption is constant ($\theta_c = \bar{\theta}_c$ where $0 < \bar{\theta}_c < 1$) in order to see how misprediction affects for a given effort, the sharing out between saving and health investment. This assumption could give sometimes a good description of the actual behaviour of people. It is abandoned in the general model.

III.3.1 Health investment and saving for a fixed effort

With $\bar{\theta}_c$ constant, the amount of effort the individual makes to improve his future either by saving or by investing in his health capital is also constant and given by $1 - \bar{\theta}_c$. The individual has just to allocate his time at the first period in a proportion θ_h to health investment, and the rest to the labour market in a proportion $\theta_s = 1 - \theta_h - \bar{\theta}_c$ to save for the second period (or to borrow if $\theta_s < 0$). By comparison with the previous model, we still have

$$\tilde{h}_1(\theta_h,\alpha_m) = h_1(\theta_h) + \alpha_m\,(\bar{h}_0 - h_1(\theta_h))$$

But now $c_0 = c_0(\bar{\theta}_c)$ is fixed and c_1 is an increasing linear function de θ_s ($dc_1/d\theta_s$ = constant)

The individual's maximization program becomes:

$$\text{Max}\,\Big|_{\theta_h,\theta_s}\quad \tilde{U}(\theta_h,\theta_s) = u(c_0(\bar{\theta}_c), \bar{h}_0) + \frac{1}{1+\delta}\,u(c_1(\theta_s), \tilde{h}_1(\theta_h))]$$

$$\text{with}\quad \bar{\theta}_c + \theta_h + \theta_s = 1 \qquad ; \quad \theta_s \geq 0 \quad \theta_h \geq 0$$

and the three (sufficient) conditions are:

$$\frac{1}{1+\delta}\,\frac{\partial\, u(c_1, \tilde{h}_1)}{\partial\, \tilde{h}_1}\,\frac{\partial \tilde{h}_1}{\partial \theta_h} - \mu^e = 0$$

$$\frac{1}{1+\delta}\,\frac{\partial\, u(c_1, \tilde{h}_1)}{\partial\, c_1}\,\partial c_1/\partial \theta_s - \mu^e = 0$$

$$-\bar{\theta}_c - \theta_h^e - \theta_s^e + 1 = 0$$

Equalizing the first two conditions shows that at the equilibrium, the individual must be indifferent at the margin between allocating one extra unit of his time to saving and health production. The impact of α_m on the equilibrium choice is more ambiguous. If we apply the same comparative static analysis as before, we obtain now[19]:

$$\frac{d\theta_h^e}{d\alpha_m} = \frac{a - b}{\det J} \qquad \text{and} \qquad \frac{d\theta_s^e}{d\alpha_m} = \frac{b - a}{\det J}$$

where the general formula for a and b are similar[20] to those found for model III.1 and III.2 and det J > 0. Because b > 0 and a has the opposite sign of the cross partial derivatives between c_1 and $\tilde{h}_1$, it follows that:

$$\frac{d\theta_h^e}{d\alpha_m} < 0 \qquad \text{and} \qquad \frac{d\theta_s^e}{d\alpha_m} > 0 \qquad\qquad \text{if} \quad u_{ch} \geq 0$$

If the marginal utility of c_1 does not depend on health, then adaptation ($\alpha > 0$), which reduces the marginal utility of health encourages the individual to increase its saving at the expense of health investment. This effect is reinforced here if the cross partial derivative is positive, because it implies that adaptation which raises the subjective level of health for a given effort also raises the marginal utility of future consumption. Returning to our concern with misprediction, it means that if the individual fails to predict correctly this adaptation, then he will *underinvest in financial assets* but *overinvest* in his health.

---

[19] The appendix B gives calculus details only for the general model but there are very close to those used here.

[20] $a = -\frac{1}{1+\delta}\,u_{ch}\,(\bar{h}_0 - h_1)$ and $b = \frac{1}{1+\delta}\frac{\partial h_1}{\partial \theta_h}\left(\frac{\partial\, u(c_1, \tilde{h}_1)}{\partial\, \tilde{h}_1} - (1-\alpha_m)\frac{\partial u(c_1, \tilde{h}_1)^2}{\partial\, \tilde{h}_1^2}\,(\bar{h}_0 - h_1)\right) > 0$

This conclusion can be inverted when the cross partial derivative is *strongly* negative so that a – b > 0. Returning to the derivatives, we can write[21] :

$$\frac{d\theta_h^e}{d\alpha_m} > 0 \quad \text{and} \quad \frac{d\theta_s^e}{d\alpha_m} < 0 \qquad \text{if } u_{ch} \text{ strongly negative}$$

In that case, *for a given effort*, adaptation still reduces per se the marginal utility of health and so the incentive to invest in it but now, it also reduces the marginal utility of future consumption and so the incentive to save. In this model however, efforts to increase future utility being constant and equal to $1-\bar{\theta}_c$, the individual can't reduce both. He must necessarily choose to reduce one and so to raise the other. As a consequence, when the cross partial derivative is significantly negative, the decrease of the marginal utility of future consumption implied by adaptation for a given effort may be strong enough sometimes to encourage the individual to save less and to increase even more his investment in health. This investment will not be sufficient however as long as the individual underestimates adaptation.

III.3.2: The general model

In the general model, θ_c is no longer fixed. The individual has now to choose to allocate his time at the first period in a proportion θ_h to health investment, and the rest to the labour market in a proportion θ_c for present consumption and in a proportion $\theta_s = 1 - \theta_h - \theta_c$ for saving (or borrowing if $\theta_s < 0$). With 3 variables, the maximization programs becomes:

$$\text{Max}\,|_{\theta_c,\theta_h,\theta_s} \quad \tilde{U}(\theta_c,\theta_h,\theta_s) = u(c_0(\theta_c), \bar{h}_0) + \frac{1}{1+\delta} u(c_1(\theta_s), \tilde{h}_1(\theta_h))]$$

$$\text{with} \quad \theta_c + \theta_h + \theta_s = 1 \quad ; \quad \theta_c \geq 0 \quad \theta_h \geq 0$$

There are now four (sufficient) conditions are:

$$\frac{\partial u(c_0, \bar{h}_0)}{\partial c_0} \frac{\partial c_0(\theta_c^e)}{\partial \theta_c} - \mu^e = 0$$

$$\frac{1}{1+\delta} \frac{\partial u(c_1, \tilde{h}_1)}{\partial \tilde{h}_1} \frac{\partial \tilde{h}_1}{\partial \theta_h} - \mu^e = 0$$

$$\frac{1}{1+\delta} \frac{\partial u(c_1, \tilde{h}_1)}{\partial c_1} \partial c_1/\partial \theta_s - \mu^e = 0$$

$$-\theta_c^e - \theta_h^e - \theta_h^e + 1 = 0$$

By deriving a last time the equilibrium variables with respect to α_m, we get the final following equations[22]:

---

[21] Indeed, the simple negativity of the cross partial derivative is not enough to guarantee this result since (a-b) positive only if for equilibrium values, given a and b,

$$-\frac{1}{1+\delta} u_{ch} (\bar{h}_0 - h_1) - \frac{1}{1+\delta} \frac{\partial h_1}{\partial \theta_h} \left(\frac{\partial u(\bar{c}_1, \tilde{h}_1)}{\partial \tilde{h}_1} - (1-\alpha_m) \frac{\partial u^2(\bar{c}_1 \tilde{h}_1)}{\partial \tilde{h}_1^2} (\bar{h}_0 - h_1) \right) > 0$$

which implies after arrangement:

$$- u_{ch} > \frac{\partial u/\partial \tilde{h}_1 . \partial h_1/\partial \theta_h}{\bar{h}_0 - h_1} - (1 - \alpha_m) \frac{\partial u^2(\bar{c}_1 \tilde{h}_1)}{\partial \tilde{h}_1^2} > 0$$

[22] Calculus details are given in appendix B

$$\frac{d\theta_c^e}{d\alpha_m} = \frac{b(\tilde{U}_{33} - \tilde{U}_{32}) + a(\tilde{U}_{22} - \tilde{U}_{32})}{\det J}$$

$$\frac{d\theta_h^e}{d\alpha_m} = \frac{\tilde{U}_{11}(a - b) - b\tilde{U}_{33} + a\tilde{U}_{32}}{\det J}$$

$$\frac{d\theta_s^e}{d\alpha_m} = \frac{\tilde{U}_{11}(b - a) - a\tilde{U}_{22} + b\tilde{U}_{32}}{\det J}$$

With $\tilde{U}_{11} < 0$, $\tilde{U}_{22} < 0$, $\tilde{U}_{33} < 0$, sign $(\tilde{U}_{23} = \tilde{U}_{32})$ = sign u_{ch}, Det $J < 0$, a and b taking identical forms as in the two previous models.

By looking carefully at the signs of each term, we can see that the direction of the change of the equilibrium variables following a marginal change in α_m is determined only if we make the hypothesis that the marginal utility of consumption (respectively health) does *not* depend on the health level (respectively consumption). In that case in effect, a = 0, hence $\tilde{U}_{32} = 0$ so that we've got:

$$\frac{d\theta_c^e}{d\alpha_m} = \frac{bU_{33}}{\det J} > 0 \quad ; \quad \frac{d\theta_h^e}{d\alpha_m} = \frac{-bU_{11} - bU_{33}}{\det J} < 0 \quad ; \quad \frac{d\theta_s^e}{d\alpha_m} = \frac{bU_{11}}{\det J} > 0$$

The signs for $d\theta_s^e/d\alpha_m$ and $d\theta_h^e/d\alpha_m$ are identical to the previous model where we assumed that θ_c was constant and where the individual was just allocating the rest of his time at the first period between health and saving. In this general model where θ_c is endogenous, we can now see that adaptation unambiguously induces the individual to raise θ_c and so to reduce his global effort towards the future. The time allocated to the labour market for saving still increases but it is more than offset by the reduction of the time allocated to health investment. Consequently, his inability to predict that adaptation will lead him to overinvest in his health at the expense of his present and future consumption.

The other result is that the consequences of a change in α_m on equilibrium variables are no longer determined when the marginal utility of consumption depends significantly positively or negatively on health capital: $d\theta_c^e/d\alpha_m$, $d\theta_h^e/d\alpha_m$ and $d\theta_s^e/d\alpha_m$ may or may not, depending on the case, be positive, null or negative. In particular, adaptation does not automatically encourage the individual to reduce his efforts toward the future. We can't be sure that $d\theta_c^e/d\alpha_m > 0$ whether u_{ch} is positive or negative. If u_{ch} is strongly positive for instance, then it may be possible for the individual to prefer to increase both future health and consumption at the expense of present consumption. To solve this indetermination, we are constrained to give more specifications to the utility function.

IV. CONCLUSION

The tendency for young people to underestimate their future capacity to adapt to old age is a very common cognitive process which should logically affect their intertemporal long-term choices. The previous reasoning is a first effort to work out its main consequences on long term saving and health investment decisions using unspecified utility function. It relies on a central double assumption: If an individual adapts at least partially to the negative consequences of old age, he will draw more utility from a same stock of youth/health when old than young but the marginal utility of health will be lower. For the individual, all happens as if, because of adaptation, the subjective level of health/youth does not decrease as fast as his objective level. Under these general assumptions, and in four small models where depending on the case, the individual allocates his time of the first period between health investment and/or the labour market for present consumption and/or saving, the three following results are obtained:

i) When individuals take their future health as given, and if the marginal utility marginal of consumption increases with health, adaptation to old age leads them to increase their saving. As long as they underestimate this adaptation though, this saving remains suboptimal and people undersave. When the marginal utility of consumption depends negatively of health, people oversave for symmetric reasons.

ii) When the individual must share out a fixed effort towards the future between health investment and financial investment (saving), then adaptation encourages the individual to increase its saving at the expense of health investment if the marginal utility of consumption does not depend on the health level. The explanation lies into the fact that adaptation reduces the marginal utility of health when old and so the incentive to invest in it during the previous period. As long as the individual underestimates his adaptation however, he does not save enough and on the contrary *overinvest* in his health. Put it simply, people spend too much time and money in cosmetic creams, surgery or sports because at the end of the day, their long-term effect for utility is not necessarily as important at people may think when they anticipate adaptation. This effect is reinforced if the cross marginal effect between consumption and health is positive and inverted when it is sufficiently negative.

iii) When people must choose during their youth between present consumption, saving and health investment, their inability to predict adaptation lead them to overinvest in their health at the expense this time of their *present* and future consumption, but only when the marginal utility of consumption does not depend on the health level. If it does significantly, the consequences of mispredicting adaptation are no longer determined under a standard but not specified utility function.

In each case, the sign and the intensity of the cross partial derivative between consumption and health in the utility function play a critical role. There are few empirical indications on the subject. Using studies on working accident, Viscusi & Evans (1990) then Evans & Viscusi (1991) indicated that the cross partial derivative is positive when health deteriorates significantly and negative when health deterioration is only marginal. If this conclusion can be extended to the youth capital as it is defined in this article, this means that we should favour the assumption that the marginal utility of consumption increases with health, at least for those who fear ageing the most. Theoretically, it also implies that the partial derivative is not independent on the idea that individuals have of their ability to adapt and that it must perhaps be introduced differently than with a simple exogenous parameter as α_m. Many other issues could be raised of course in particular about the timing of adaptation and the need to treat it through a life-cycle model. To go further, some of the conclusions of the previous models have also to be tested empirically.

Whatever the possible future developments, the “myth of a sad old age” as well as the cognitive processes that underlie it are well-established facts. The previous models showed that they can amplify or correct the effects of other cognitive biases identified in the literature like an unrealistically high discount rate or time-inconsistent preference for immediate gratification (Rabin and Donohue, 1999 for instance). These conclusions need to be explored beyond this first work.

Appendix

A. Note on an alternative formalization of mispredicted adaptation: Deformation of the utility function

As indicated in the main text, adaptation and its underestimation can alternatively be formalized as a deformation of the utility function. To model adaptation by using the same kind of linear indicator, let us consider the following specification:

$$u_1(c_1,h_1) = \alpha u_0(c_1,h_0) + (1-\alpha)\, u_0(c_1, h_1) \qquad \text{with } \alpha \in [0,1] \text{ and } h_1 < h_0$$

and $u_1(c_1,0) = 0$ since adaptation exists as long as the individual is alive.

α is the indicator of adaptation. When $\alpha = 0$ (no adaptation), $u_1(c_1,h_1) = u_0(c_1,h_1)$. In the extreme and unrealistic case of full adaptation (α=1), the level of health capital does not matter any longer for utility and $u_1(c_1,h_1) = u_0(c_1,h_0)$ when $0 < h_1 < h_0$.

Since by hypothesis, $h_1 < h_0$, for $\alpha > 0$, we've got:

$$u_1(c_1,h_1) > u_0(c_1,h_1) \qquad \text{(i)}$$

and looking at the marginal utility in presence of adaptation, $\partial u_1/\partial h_1 = (1-\alpha)\partial u_0/\partial h_1$, we conclude that

$$\partial u_1/\partial h_1 < \partial u_0/\partial h_1 \qquad \text{(ii)}$$

(i) and (ii) are the general standard desired properties to describe adaptation postulated in II.2

Following Loewenstein, O'Donoghue and Rabin [2003], we can also model the underestimation of this adaptation as a projection bias with a simple linear specification by assuming that the person mispredicts her future utility as soon as there exists a constant $m \in\ ]0,1]$ such that:

$$\tilde{u}_1(c_1,h_1) = m\,.u_0(c_1,h_0) + (1-m)\, u_1(c_1,h_1) \qquad \text{for } h_1 < h_0$$

where $\tilde{u}_1(c_1,h_1)$ represents the utility predicted at period 0 for period 1.

By substituting u_1 in the previous equation by its expression related to α, we obtain:

$$\tilde{u}_1 = m\, u_0(c_1, h_0) + (1-m)\, [\alpha u_0(c_1,h_0) + (1-\alpha)\, u_0(c_1, h_1)]$$

and after rearranging: $\tilde{u}_1 = \alpha_m\, u_0(c_1,h_0) + (1-\alpha_m)\, u_0(c_1, h_1)$

where $\alpha_m = \alpha - m\alpha$ is the *predicted level of adaptation.*

After introducing a constant positive discount rate δ, the predicted total utility of the person over the two periods calculated at time 0 is then given by

$$\tilde{U} = u_0(c_0, h_0) + \frac{1}{1+\delta}\tilde{u}_1(c_1,h_1)$$

$$\text{With} \quad \tilde{u}_1(c_1,h_1) = \alpha_m\, u_0(c_1,h_0) + (1-\alpha_m)\, u_0(c_1, h_1)$$

In the central text, we supposed instead to model mispredicted adaptation that all happens as if the subjective level of health (or perceived age), noted h_1^{α}, does not decrease as fast as his objective level (biological age). In this case, we ended up with the following specification:

$$\tilde{U} = u(c_0, h_0) + \frac{1}{1+\delta} u(c_1, \tilde{h}_1)$$

$$\text{With} \quad \tilde{h}_1 = h_1 + \alpha_m\, (\bar{h}_0 - h_1)$$

Calculus available under request show that applying the same comparative statics to the equilibrium gives exactly the same general formula for the derivatives of the equilibrium variables with respect to α_m in all models and so the same results. Thus the conclusions drawn in the general text do not depend on our "subjective level of health" hypothesis.

B. Derivation of the 4 equilibrium conditions in the general model (III.3.2)

The four equilibrium conditions of the general model (III.3.2) are given by:

$$\frac{\partial u(c_0, \bar{h}_0)}{\partial c_0}\frac{\partial c_0(\theta_c^e)}{\partial \theta_c} - \mu^e \qquad = 0$$

$$\frac{1}{1+\delta}\frac{\partial u(c_1,\tilde{h}_1)}{\partial \tilde{h}_1}\frac{\partial \tilde{h}_1}{\partial\theta_h} - \mu^e = 0$$

$$\frac{1}{1+\delta}\frac{\partial u(c_1,\tilde{h}_1)}{\partial c_1}\partial c_1/\partial\theta_s - \mu^e = 0$$

$$-\theta_c^e - \theta_h^e - \theta_h^e + 1 = 0$$

By deriving the equilibrium variables with respect to α_m, we've got:

- $$\frac{\partial}{\partial\alpha_m}\left[\frac{\partial u(c_0(\theta_c^e(\alpha_m)),\bar{h}_0)}{\partial\theta_c}\right] - \frac{d}{d\alpha_m}\mu^e(\alpha_m) = 0$$

$$\rightarrow \left[\frac{\partial^2 u(c_0,\bar{h}_0)}{\partial c_0^2}\left(\frac{\partial c_0(\theta_c^e(\alpha_m))}{\partial\theta_c}\right)^2 + \frac{\partial u(c_0,\bar{h}_0)}{\partial c_0}\frac{\partial^2 c_0(\theta_c^e(\alpha_m))}{\partial\theta_c^2}\right]\frac{d\theta_c^e}{d\alpha_m} - \frac{d\mu^e}{d\alpha_m} = 0$$

$$\rightarrow \tilde{U}_{11}\frac{d\theta_c^e}{d\alpha_m} - \frac{d\mu^e}{d\alpha_m} = 0 \qquad \textbf{(1)}$$

- $$\frac{\partial}{\partial\alpha_m}\left[\frac{1}{1+\delta}\frac{\partial u(c_1,\tilde{h}_1)\partial h_1(\theta_h^e)}{\partial\tilde{h}_1\,\partial\theta_h}(1-\alpha_m)\right] - \frac{d\mu^e}{d\alpha_m} \equiv 0$$

$$\rightarrow \frac{\partial}{\partial\alpha_m}\frac{1}{1+\delta}\left(\frac{\partial u(c_1,\tilde{h}_1)}{\partial\tilde{h}_1}\right)\frac{\partial\tilde{h}_1}{\partial\theta_h} + \frac{1}{1+\delta}\frac{\partial u}{\partial\tilde{h}_1}\frac{\partial}{\partial\alpha_m}\left(\frac{\partial\tilde{h}_1}{\partial\theta_h}\right) - \frac{d\mu^e}{d\alpha_m} = 0$$

$$\rightarrow \frac{1}{1+\delta}\frac{\partial^2 u}{\partial\tilde{h}_1^2}.\left(\frac{\partial\tilde{h}_1}{\partial\theta_h}\frac{\partial\theta_h^e}{\partial\alpha_m} + (\bar{h}_0 - h_1)\right)\frac{\partial\tilde{h}_1}{\partial\theta_h} + \frac{1}{1+\delta}\frac{\partial^2 u}{\partial c_1\partial\tilde{h}_1}\frac{\partial c_1\partial\theta_s^e}{\partial\theta_s\partial\alpha_m} + \frac{1}{1+\delta}\frac{\partial u}{\partial\tilde{h}_1}\left(-\frac{\partial h_1}{\partial\theta_h} + (1-\alpha_m)\frac{\partial^2\tilde{h}_1}{\partial\theta_h^2}.\frac{\partial\theta_h^e}{\partial\alpha_m}\right) - \frac{d\mu^e}{d\alpha_m} = 0$$

$$\rightarrow \frac{1}{1+\delta}\left(\frac{\partial^2 u}{\partial\tilde{h}_1^2}\left(\frac{\partial\tilde{h}_1}{\partial\theta_h}\right)^2.\frac{\partial\theta_h^e}{\partial\alpha_m} + \frac{\partial u}{\partial\tilde{h}_1}\frac{\partial^2\tilde{h}_1}{\partial\theta_h^2}\right).\frac{\partial\theta_h^e}{\partial\alpha_m} + \frac{1}{1+\delta}\frac{\partial^2 u}{\partial c_1\partial\tilde{h}_1}\frac{\partial c_1\partial\theta_s^e}{\partial\theta_s\partial\alpha_m} + \frac{1}{1+\delta}\frac{\partial^2 u\,\partial\tilde{h}_1}{\partial\tilde{h}_1^2\partial\theta_h}(\bar{h}_0 - h_1) - \frac{1}{1+\delta}\frac{\partial u}{\partial\tilde{h}_1}\frac{\partial h_1}{\partial\theta} - \frac{d\mu^e}{d\alpha_m} = 0$$

$$\rightarrow \tilde{U}_{22}\frac{\partial\theta_h}{\partial\alpha_m} + \tilde{U}_{32}\frac{\partial\theta_s^e}{\partial\alpha_m} - \frac{d\mu^e}{d\alpha_m} = \frac{1}{1+\delta}\frac{\partial h_1}{\partial\theta_h}\left(\frac{\partial u}{\partial\tilde{h}_1} - (1-\alpha_m)\frac{\partial u^2}{\partial\tilde{h}_1^2}(\bar{h}_0 - h_1)\right) \qquad \textbf{(2)}$$

- $$\frac{\partial}{\partial\alpha_m}\left[\frac{1}{1+\delta}\frac{\partial u(c_1,\tilde{h}_1)}{\partial c_1}\frac{\partial c_1}{\partial\theta_s}\right] - \frac{d\mu^e}{d\alpha_m} = 0$$

$$\rightarrow \frac{1}{1+\delta}\frac{\partial}{\partial\alpha_m}\left(\frac{\partial u(c_1,\tilde{h}_1)}{\partial c_1}\frac{\partial c_1}{\partial\theta_s}\right) + \frac{\partial}{\partial\alpha_m}\frac{1}{1+\delta}\left(\frac{\partial u}{\partial c_1}\right)\frac{\partial c_1}{\partial\theta_s} + \frac{\partial u}{\partial c_1}\frac{1}{1+\delta}\frac{\partial}{\partial\alpha_m}\left(\frac{\partial c_1}{\partial\theta_s}\right) - \frac{d\mu^e}{d\alpha_m} = 0$$

$$\rightarrow \frac{1}{1+\delta}\frac{\partial^2 u}{\partial c_1^2}\left(\frac{\partial c_1}{\partial\theta_h}\right)^2\frac{d\theta_s^e}{d\alpha_m} + \frac{1}{1+\delta}\frac{\partial^2 u}{\partial\tilde{h}_1\partial c_1}\frac{\partial c_1}{\partial\theta_s}\frac{\partial\tilde{h}_1}{\partial\theta_h}\frac{d\theta_h^e}{d\alpha_m} + \frac{1}{1+\delta}\frac{\partial^2 u}{\partial\tilde{h}_1\partial c_1}(\bar{h}_0 - h_1) - \frac{d\mu^e}{d\alpha_m} = 0$$

$$\rightarrow \frac{1}{1+\delta}\left(\frac{\partial^2 u}{\partial c_1^2}\left(\frac{\partial c_1}{\partial\theta_h}\right)^2 + \frac{1}{1+\delta}\frac{\partial u}{\partial c_1}\frac{\partial^2 c_1}{\partial\theta_s^2}\right)\frac{d\theta_s^e}{d\alpha_m} + \frac{1}{1+\delta}\frac{\partial^2 u}{\partial\tilde{h}_1\partial c_1}\frac{\partial c_1}{\partial\theta_s}\frac{\partial\tilde{h}_1}{\partial\theta_h}\frac{d\theta_h^e}{d\alpha_m} - \frac{d\mu^e}{d\alpha_m} + \frac{1}{1+\delta}\frac{\partial^2 u}{\partial\tilde{h}_1\partial c_1}(\bar{h}_0 - h_1) - \frac{d\mu^e}{d\alpha_m} = 0.$$

$$\rightarrow \tilde{U}_{32}\frac{d\theta_h^e}{d\alpha_m} + \tilde{U}_{33}\frac{d\theta_s^e}{d\alpha_m} - \frac{d\mu^e}{d\alpha_m} = -\frac{1}{1+\delta}\frac{\partial^2 u}{\partial\tilde{h}_1\partial c_1}(\bar{h}_0 - h_1) \qquad \textbf{(3)}$$

With

$$\tilde{U}_{11} = \frac{\partial^2 u(c_0,\bar{h}_0)}{\partial c_0^2}\left(\frac{\partial c_0(\theta_c^e)}{\partial\theta_c}\right)^2 + \frac{\partial u(c_0,\bar{h}_0)}{\partial c_0}\frac{\partial^2 c_0(\theta_c^e)}{\partial\theta_c^2} < 0$$

$$\tilde{U}_{22} = \frac{1}{1+\delta}\left(\frac{\partial u^2(c_1\,\tilde{h}_1)}{\partial\tilde{h}_1^2}\left(\frac{\partial\tilde{h}_1(\theta_h^e)}{\partial\theta_h}\right)^2 + \frac{\partial u(c_1\,\tilde{h}_1)}{\partial\tilde{h}_1}\frac{\partial^2\tilde{h}_1(\theta_h^e)}{\partial\theta_h^2}\right) < 0$$

$$\tilde{U}_{33} = \frac{1}{1+\delta}\frac{\partial^2 u(c_1,\tilde{h}_1)}{\partial c_1^2}\left(\frac{\partial c_1(\theta_s^e)}{\partial\theta_s}\right)^2 + \frac{\partial u(c_1,\tilde{h}_1)}{\partial c_1}\frac{\partial^2 c_1(\theta_s^e)}{\partial\theta_s^2} < 0$$

$$\tilde{U}_{32} = \tilde{U}_{23} = \frac{1}{1+\delta}\frac{\partial^2 u(c_1,\tilde{h}_1)}{\partial\tilde{h}_1\partial c_1}\frac{\partial c_1(\theta_c^e)}{\partial\theta_s}\frac{\partial\tilde{h}_1(\theta_h^e)}{\partial\theta_h}$$

$$\bullet\ -\frac{d\theta_c^e}{d\alpha_m} - \frac{d\theta_h^e}{d\alpha_m} - \frac{d\theta_s^e}{d\alpha_m} = 0 \qquad \textbf{(4)}$$

Equations (1 $\rightarrow$ 4) constitutes a four-equation system which can be presented under a matrix form:

$$\begin{pmatrix} \tilde{U}_{32} & 0 & 0 & -1 \\ 0 & \tilde{U}_{22} & \tilde{U}_{32} & -1 \\ 0 & U_{32} & U_{33} & -1 \\ -1 & -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} d\theta_c^e/d\alpha_m \\ d\theta_h^e/d\alpha_m \\ d\theta_s^e/d\alpha_m \\ d\mu^e/d\alpha_m \end{pmatrix} = \begin{pmatrix} 0 \\ b \\ a \\ 0 \end{pmatrix}$$

Using Cramer rule gives:

$$\frac{d\theta_c^e}{d\alpha_m} = \frac{\det \begin{pmatrix} 0 & 0 & 0 & -1 \\ b & \tilde{U}_{22} & \tilde{U}_{32} & -1 \\ a & U_{32} & U_{33} & -1 \\ 0 & -1 & -1 & 0 \end{pmatrix}}{\det J} = \frac{b(\tilde{U}_{33} - \tilde{U}_{32}) + a(\tilde{U}_{22} - \tilde{U}_{32})}{\det J}$$

$$\frac{d\theta_h^e}{d\alpha_m} = \frac{\det \begin{pmatrix} \tilde{U}_{11} & 0 & 0 & -1 \\ 0 & b & \tilde{U}_{32} & -1 \\ 0 & a & U_{33} & -1 \\ -1 & 0 & -1 & 0 \end{pmatrix}}{\det J} = \frac{\tilde{U}_{11}\,(a - b) - b\tilde{U}_{33} + a\tilde{U}_{32}}{\det J}$$

$$\frac{d\theta_s^e}{d\alpha_m} = \frac{\det \begin{pmatrix} \tilde{U}_{11} & 0 & 0 & -1 \\ 0 & \tilde{U}_{22} & b & -1 \\ 0 & U_{32} & a & -1 \\ -1 & -1 & 0 & 0 \end{pmatrix}}{\det J} = \frac{\tilde{U}_{11}\,(b - a) - a\tilde{U}_{22} + b\tilde{U}_{32}}{\det J}$$

References

Afsa Cédric, Marcus Vincent, (2008) Le Bonheur attend-il le nombre des années ? in *France, portrait social, édition 2008*. Insee.

Argyle Michael, (2001) *The psychology of happiness*. Routledge: London, Second edition.

Blanchflower David G., Oswald Andrew J., (2006) Is Well-Being U-Shaped Over the Life Cycle ? February, *NBER Working Paper* No W12935.

Brickman Phillip, Coates Dan, Janoff-Bulman Ronie, (1978) Lottery winners and accident victims: is happiness relative ? *Journal of Personality and Social Psychology*, 36(8), 917-927.

Carstensen Laura L., Isaacowitz Derek M., Charles Suzan T., (1999). Taking time seriously: A theory of socioemotional selectivity. *American Psychologist*, 54, 165-181.

Charles Suzan T., Mather Mara, Carstensen Laura L., (2003). Aging and Emotional Memory: The Forgettable Nature of Negative Images for Older Adults. *Journal of Experimental Psychology:*, 132(2), 310-324.

Costa Jr Paul T., Zonderman Alan B., McCrae, R.R., Cornoni-Huntley, J., Locke, B.Z., Barbano, H E., (1987) Longitudinal Analyses of Psychological Well-Being in a National Sample: Stability of Mean Levels. *Journal of Gerontology*, 42, 50-55.

Demakakos Panayotes, Hacker Elizabeth, Gjonça Edlira (2006) Perceptions of aging in *Retirement, health and relationships of the older population in England. The 2004 English Longitudinal Study of Ageing (wave 2).* Banks James, Breeze Elizabeth, Carli Lessof & Nazroo James (editors). The Institute for Fiscal Studies. London.

Diener Ed, Lucas Richard E., Scollon Christie Napa (2006) Beyond the Hedonic Treadmill, *American Psychologist*, 61(4), 305-314.

Eastwick Paul W., Finkel Eli J., Krishnamurti Tamar, Loewenstein George, (2007) Mispredicting Distress Following Romantic Breakup: Revealing the time course of the affective forecasting error. *Journal of Experimental Social Psychology, September*

Evans William N., Viscusi Kip W., (1991) Estimation of State Dependant Utility Functions Using Survey Data. *Review of Economics and Statistics*, 73(1), 94-104.

Fung Hélène H., Carstensen, Laura L., (2003) Sending memorable messages to the old: Age differences in preferences and memory for advertisements. *Journal of Personality and Social Psychology*, 85(1), 163-178.

Gilbert Daniel T.,, Pinel E. C., Wilson T., D., Blumberg S., J., Wheatley T. P., (1998) Immune Neglect: A Source of Durability Bias in Affective Forecasting. *Journal of Personality and Social Psychology*, 75, 617-638.

Grossman Michael, (1972) On the Concept of Health Capital and the Demand for Health. *Journal of Political Economy*, 80, 223-255.

Johannesson Magnus, Gerdtham Ulf-G, (1997) The relationship between happiness, health and socio-economic factors: results based on Swedish micro data., Department of Economics, Stockholm School of Economics, Stockholm, Sweden *Working Paper Series in Economics and Finance* No. 207 November 1997.

Kimball Miles S., (1990) Precautionary Saving in the Small and in the Large. *Econometrica*, 58 (1), 53-73.

Knapp James L., Stubblefield Patricia, (2000) Changing students' perceptions of aging: the impact of an intergenerational service learning course. *Educational Gerontology*, 26, 611–621.

Lacey Heather P., Fagerlin, A., Loewenstein George, Smith Dylan M., Riis J., and Ubel Peter A. (2006). It must be awful for them: Healthy people overlook disease variability in quality of life judgments. *Judgment and Decision Making*, 1(2), 146-152.

Lacey Heather P., Smith Dylan M., Ubel Peter A., (2006) Hope I Die Before I Get Old: Mispredicting Happiness Across the Adult Lifespan. *Journal of Happiness Studies*, 7(2), 167-182.

Loewenstein George (2005) Projection bias in medical decision-making. *Medical Decision Making*, 25(1), 96- 105.

Loewenstein George, O'Donoghue Ted and Rabin Matthew, (2003) Projection Bias in Predicting Future Utility. *Quarterly Journal of Economics*, 118(4), 1209–48.

Lykken David T., Tellegen Auke, (1996) Happiness Is a Stochastic Phenomenon. *Psychological Science*, 7, 186-189.

Mroczek Daniel K, Kolarz Christian M., (1998) The effect of age on positive and negative affect: A developmental perspective on happiness. *Journal of Personality and Social Psychology*, 75, 1333-1349.

Mroczek Daniel K, Spiro Avron, (2005) Changes in Life Satisfaction during Adulthood: Findings from the Veterans Affairs Normative Aging Study. *Journal of Personality and Social Psychology*, 88(1), 189-202.

O'Donoghue Edouard D., Rabin Matthew, (1999) Procrastination in Preparing for Retirement in *Behavioral Dimensions of Retirement Economics*, Henry Aaron, editor, The Brookings Institution.

Oswald Andrew J., Powdthavee Nattavudh (2008) Does happiness adapt? A longitudinal study of disability with implications for economists and judges. *Journal of Public Economics*. Forthcoming.

Palmore Erdman B., (1998) *The facts on aging quiz.* New York. Springer.

Ross Catherine E., Mirowksy John, (2008) Age and the balance of emotions. *Social Science and Medecine*, 66(12), 2391-2400.

Ubel Peter A., Loewenstein George, Schwarz Norbert and Smith Dylan M., (2005). Misimagining the unimaginable: The happiness gap and healthcare decision making. *Health Psychology*, 24(4), S577-S62.

Viscusi Kip W., Evans William N., (1990) Utility Functions That Depend on Health Status: Estimates and Economic Implications. *American Economic Review*, 80 (3), 353-374.

Yang Yang, (2008) Social Inequalities in Happiness in the United States, 1972 to 2004: An Age-Period Cohort Analysis. *American Sociological Review*, 73, 204-226.