Welfare Economic Foundations of Health Status Measures

Jens Leth Hougaard   Hans Keiding
University of Copenhagen

July 2002

Abstract

Measures of health status, such as e.g. the QALY (Quality Adjusted Life Years) measure, have been proposed as tools in the economic assessment of medical interventions. There is by now an extensive literature on the possible foundations in individual utility theory of such measures. The present paper focuses on problems related to aggregation over different types or aspects of health. We address this problem by considering a general equilibrium model of a society, where different aspects of health are formalized as Lancasterian characteristics. We consider the welfare theoretical basis for evaluation of changes in allocation. In particular, we investigate the conditions under which individuals will have the same marginal rates of substitution between health characteristics, shown to be a precondition for a meaningful measurement of health status. It turns out that this equality is attained under conditions of separability either of individual health production or of preferences.

Keywords: Health status measures, QALYs, cost-effectiveness analyses, consumers' characteristics, general equilibrium.

JEL-classification: D6, I0.

1 Introduction

In recent years, the growth of health related expenditure in most countries has led to an increased concern for setting the right priorities and for choosing the right methods to evaluate medical interventions. Since output produced in the health care sector is not directly marketed, and the final goal of the sector's production, namely health, is not immediately amenable to precise measurement, investment analysis is not easily applied to medical technologies and treatments. It is not surprising, therefore, that there have been many attempts to measure health status or changes in health status. Among the many suggestions to measure health status the most prominent is the Quality Adjusted Life Years (QALY) index which is widely discussed in the literature (cf. e.g. Torrance, 1986; Torrance and Feeny, 1989; Loomes and McKenzie, 1989; Bleichrodt, Wakker and Johannesson, 1997; Bleichrodt and Quiggin, 1997; see also the recent survey in the survey in Dolan, 2000).

Several authors have remarked that the use of a health index in cost-effectiveness analyses involves aggregation across individuals. However, there is also another kind of aggregation involved; if 'health' is not a well-defined one-dimensional quantity but has several aspects or components (ability to move around, ability to hear, to see, etc.), then the comparison of medical treatments according to QALYs gained per dollar spent presupposes that the QALY gain of, say, improved ability to see, relative to the QALY gain of some other kind of improved health, for example ability to hear, should be the same for all individuals. Technically, marginal rates of substitution between different aspects of health should not differ among individuals, even if their individual utilities and society's weighing of these individuals may differ.

This equality of marginal rates of substitution (in different types of health) among individuals (explained in some more detail in the following section) is an assumption which is much more restrictive than what economists are willing to admit when dealing with consumption of ordinary commodities. Here it is usually taken for granted that individuals have different preferences and therefore also different marginal rates of substitution between goods. Similarly, it would seem reasonable to expect that individuals evaluate different aspects of health in different ways (a professional musician may indeed be more worried about the ability to hear than the ability to move around, and a professional athlete might well take the opposite view).

As is well-known, individual marginal rates of substitution between goods become identical when the goods can be traded in a market. Now this intervention of the market does not work immediately with respect to different aspects of health, since they cannot be traded. However, since health is obtained by the individual through an overall decision that also involves goods obtained at the market, it is conceivable that the market may result in uniformity of marginal rates of substitution also for the various aspects of health. This is what is investigated in the paper, where health components are treated as Lancasterian consumption characteristics, and indeed equality of marginal rates of substitution holds in some, unfortunately rather restrictive cases, characterized by separability in the 'production' of health from commodities or in preferences.

The paper is structured as follows: Section 2, provides a more detailed discussion of the specific aggregation problem involved in the use of health measures of the QALY type in cost-effectiveness analysis. In Section 3, we introduce our basic general equilibrium model with health characteristics, and the following Section 4 contains two simple examples showing what is at stake. In Section 5, we assume that individuals differ in preferences but have access to the same "household" technology (i.e. ways of producing health from commodities). We introduce a separability condition on household technologies and show that equality of marginal rates of substitution obtains if and only if the separability condition is satisfied. Section 6 treats the situation where preferences are identical but where household technologies vary among individuals, and a similar characterization by separability, this time of preferences, is obtained. The concluding section 7 contains final remarks; in particular, we return to the initial problem of health measurement and interpret our results in this context. The proofs of the theorems in the text have been collected in an appendix.

2 Health measures in cost-effectiveness analysis and the need for aggregation

In this section we shall briefly discuss the role of health status measures in cost-effectiveness analysis. Our approach to cost-effectiveness is that of welfare economics (for a discussion the reader is referred to Gold et al., 1996; Garber, 2000; Brouwer and Koopmanschap, 2000). Thus, we consider a society with, say, m consumers, deriving utility from the consumption of l commodities as well as from the enjoyment of health in its, say, $L > 1$ different aspects. The assumption that there are several different aspects of health to be measured separately fits with intuition and it also corresponds to the approach in practical health status measurements (such as e.g. SF-36, cf. Ware, 1995). For a more detailed descriptions of definite aspects of health, cf. e.g. Sintonen (1981).

We assume that consumers have utility functions $u_i(x_i, h_i)$ defined on consumption of commodities and health characteristics, where x_i and h_i are vectors of consumption of commodities and health, respectively. Now a medical treatment can be formalized as a displacement $(dx_i, dh_i)_{i=1}^m$ of the current allocation of commodities and individual enjoyment of health for each individual.

As usual in the theoretical cost-benefit literature (see for example Lessourne (1975) and Laffont (1988)), we assume that society's preferences over alternative allocations of commodities and health can be described by a social welfare function

$$S(u_1(x_1, h_1), \ldots, u_m(x_m, h_m)).$$

Then the change in society's welfare associated with the displacement $(dx_i, dh_i)_{i=1}^m$ in health and commodity consumption is

$$dS = \sum_{i=1}^m S_i' \sum_{j=1}^l u_{ij}' dx_{ij} + \sum_{i=1}^m S_i' \sum_{j=1}^L u_{ij}' dh_{ij}.$$

Note that everything except for the displacements is unknown to the analyst. However, if we assume that (i) commodities are bought in a market at prices p_j, and (ii) the income distribution in society is optimal for given health, then the first term reduces to

$$K \sum_{j=1}^l p_j \sum_{i=1}^m dx_{ij};$$

indeed, substituting from the consumers' first order conditions $\lambda_i p_j = u_{ij}'$ for utility maximization at given prices and incomes which hold due to (i), the term takes the form $\sum_{j=1}^l S_i' \lambda_i p_j \sum_{i=1}^m dx_{ij}$. Since the Lagrangian multiplier λ_i is consumer i's marginal utility of income, the assumption (ii) of optimal income distribution then implies that $S_i' \lambda_i$ is independent of i and equals some $K > 0$.

In order to perform a similar reduction of the second term related to enjoyment of health we need assumptions which are counterparts of (i) and (ii). Assume therefore that (i') individual utility depends on commodity and health consumption via a health index, i.e. there is a function Q such that

$$u_i(x_i, h_i) = \tilde{u}_i(x_i, Q(h_i))$$

for all i. Then $u_{ij}' = \tilde{u}_{i,l+1}' Q_j'$ for each i, so that u_{ij}' is a product of a term depending only on i and one depending only on j; in particular, the marginal rate of substitution $u_{i,j}'/u_{i,k}' = Q_j'/Q_k'$ is the same for all individuals. Assuming in addition that (ii') the distribution of the values of Q is considered optimal, so that $S_{ij}' \tilde{u}_{i,l+1}' = S_{i'j}' \tilde{u}_{i',l+1}'$ for all consumers i and i', we get the expression

$$dS = Kp \cdot \sum_{i=1}^m dx_i + M \sum_{i=1}^m dq_i, \tag{1}$$

where for each i, $dq_i = \sum_{j=1}^L Q_j' dh_{ij}$, is the displacement of the health index experienced by individual i. This implies that evaluation of the desirability of medical treatments for society reduces to the assessment of pairs

$$(p \cdot \sum_{i=1}^m dx_i, \sum_{i=1}^m dq_i) \tag{2}$$

according to a linear criterion with unknown weights K and M.

If both terms in (2) are strictly positive, society clearly gains in welfare ($dS > 0$) by adopting the new treatment, so the interesting case is where one of the terms, typically the first, is negative. In the latter case, we may perform comparisons of two treatments (even without knowledge of the constants K and M): If $(p \cdot \sum_{i=1}^m dx_i^0, \sum_{i=1}^m dq_i^0)$ $(p \cdot \sum_{i=1}^m dx_i^1, \sum_{i=1}^m dq_i^1)$ are the data of two alternative treatments, then

$$Kp \cdot \sum_{i=1}^m dx_i^1 + M \sum_{i=1}^m dq_i^1 > Kp \cdot \sum_{i=1}^m dx_i^0 + M \sum_{i=1}^m dq_i^0$$

if and only if

$$\frac{p \cdot \sum_{i=1}^m dx_i^1}{\sum_{i=1}^m dq_i^1} < \nu \frac{p \cdot \sum_{i=1}^m dx_i^0}{\sum_{i=1}^m dq_i^0}$$

for some $\nu > 0$. Thus, foregone consumption per unit of health gained should be smaller in treatment 1 than in treatment 0 (under the additional assumption that treatments may be introduced on an arbitrary scale, which accounts for the constant ν in the above expression).

As is seen, in order to perform cost-effectiveness analysis, we need to aggregate over different aspects of health; the crucial assumption above is (i') which boils down to an assumption of equality among individuals of marginal rates of substitution between different aspects of health. If we can point to an underlying reason why marginal utilities of all the different types of health become identical across individuals, then cost-effectivness analysis with health status measurement would be as sound as traditional cost-benefit in a world with no health-related outcomes.

In the following sections, we investigate whether the presence of a market, which of course by necessity must be restricted to ordinary commodities, can give us not only (i), but also (i'), as a result of optimizing behavior of individuals under given institutional constraints. As a matter of fact, it can, but only under rather restrictive assumptions, and the nature of these assumptions will be disclosed as we proceed.

3 A general equilibrium model with consumer characteristics

In order to see whether the presence of a market in ordinary commodities may lead to common evaluation of the relative importance of different aspects of health, we consider a general equilibrium model with consumption characteristics in the sense of Lancaster (1971). In our model below, consumers are mainly concerned about their bundle of characteristics (related to health as well as to other aspects of life), and commodities are only important insofar as they can be used as inputs in the individual's own ("household") production of characteristics. This approach to health consumption using Lancaster's (1971) consumer characteristics is not new in health economics, see e.g. the intertemporal health consumption model of Grossman (1972). Moreover, the connection between QALYs and Lancaster's characteristics was also noticed by Williams (1985) and Culyer (1990).

Thus, consider an economy $\mathcal{E}$ with l commodities. Consumers $i \in \{1, \dots, m\}$ buy commodity bundles $x_i \in \mathbf{R}_+^l$ in the market and transform these commodity bundles into bundles ξ_i of L different consumers' characteristics which are used for final consumption. Each consumer i has a utility function u_i defined on bundles of characteristics, that is on $\mathbf{R}_+^L$. We have already encountered characteristics in the previous section as the different aspects of health $h_{i1}, \dots, h_{iL}$ important for individual i; in a more general setting, there may of course be other characteristics not necessarily related to health. For simplicity of notation, we assume that utility is defined only on characteristics; this is no restriction as we may always consider each ordinary commodity as another characteristic.

To obtain the characteristics bundle ξ_i, the consumer uses a household technology $T_i \subset \mathbf{R}_+^l \times \mathbf{R}_+^L$ transforming commodity bundles into characteristics bundles; a household production is a pair (x_i, ξ_i) belonging to T_i. We may think of the household technology as a set of 'blueprints' for health-seeking behavior, specifying which commodities (food, beverages, medical services) to buy in order to achieve a desired enjoyment of health. Here the term "household" is in accordance with the terminology of Lancaster, but when dealing with technologies for transforming marketable commodities into individual health, it will of course go far beyond the realm of the household. In our application of the model, medical treatments, using services of professionals as well as individual inputs of time, will be the most important components of the household technology, but it will of course also contain all other procedures for getting better health out of goods and services which are available to the consumer.

Finally, we need to introduce production of ordinary commodities in the model, but since this is not an important part of our investigation, we shall treat commodity production (as distinct from household production) in a very summary way: There is a given set $Y \subset \mathbf{R}_+^l$ of feasible aggregate commodity bundles in the economy $\mathcal{E}$.

In the subsequent analysis, we shall occasionally make use of the following assumption, stating that the economy considered satisfies the usual rather weak conditions of well-behavedness:

Assumption 1 The economy $\mathcal{E}$ satisfies:

(i) For each i, u_i is continuous, weakly monotonic (i.e. $\xi_i' > \xi_i$ implies that $u_i(\xi_i') > u_i(\xi_i)$), and quasi-concave.

(ii) For each i, T_i is closed, convex, monotonic (i.e. for all $(x_i, \xi_i) \in T_i$, if $x_i' \geq x_i$, $x_i' \neq x_i$, then there is $\xi_i' \geq \xi_i$, $\xi_i' \neq \xi_i$ with $(x_i', \xi_i') \in T_i$) and relevant (i.e. if $(x_i, \xi_i) \in T_i$ and $\xi_i \geq 0$, $\xi_i \neq 0$, then $x_i \geq 0$, $x_i \neq 0$). Moreover, T_i satisfies free disposability (i.e. if $(x_i, \xi_i) \in T_i$, and if $x_i' \geq x_i$, $\xi_i' \leq \xi_i$, then $(x_i', \xi_i') \in T_i$).

(iii) The set Y satisfies free disposability and contains 0 in its interior.

An *allocation* in the economy $\mathcal{E}$ is a pair (x, ξ), where $x = (x_i)_{i=1}^m$ is a commodity allocation and $\xi = (\xi_i)_{i=1}^m$ a characteristics allocation. It is *individually feasible* if

$$x_i \in \mathbf{R}_+^l, \ (x_i, \xi_i) \in T_i \text{ for each } i$$

(so that the specified characteristics bundles can be obtained in the household technologies with the specified commodity bundles), and *aggregate feasible* if $\sum_{i=1}^m x_i \in Y$; it is *feasible* if it is both individually and aggregate feasible. An allocation (x, ξ) in $\mathcal{E}$ is *Pareto optimal* if it is feasible and if there is no other feasible allocation (x', ξ') such that $u_i(\xi_i') \geq u_i(\xi_i)$ for all i with at least one strict inequality.

Since our aim is to investigate whether market behavior by individuals has consequences for marginal rates of substitution for characteristics, we shall consider equilibria with respect to a market (for commodities). The following definition is a straightforward extension of a market equilibrium for economies with only ordinary commodities to our present case: An *equilibrium* in the economy $\mathcal{E}$ is an array (x, ξ, p), where (x, ξ) is a feasible allocation, and $p \in \mathbf{R}_+^l$ is a price system for commodities, such that consumers maximize utility and producers maximize profits. For the consumers $i = 1, \ldots, m$, this means that ξ_i maximizes u_i on

$$\{\xi_i'' \mid \exists (x_i', \xi_i') \in T_i,\ p \cdot x_i' \leq p \cdot x_i\},$$

so that consumers maximize utility under the constraints given by income and household technology: If there is some commodity bundle which inserted into the household technology would yield a better characteristics bundle than the equilibrium commodity bundle, then this commodity bundle is too expensive at the equilibrium prices.

Since we are interested in individual marginal utilities of characteristics, it will be convenient have these these marginal utilities or individual shadow prices appear explicitly. Formally, we may define a *market equilibrium with characteristics prices* (shorthand: an equilibrium) as an array (x, ξ, p, q), where (x, ξ) is a feasible allocation, p a price system on commodities, and $q = (q_i)_{i=1}^m$ a family of (individual) price vectors $q_i \in \mathbf{R}_+^L$ on characteristics, such that

(i) for each consumer i, ξ_i maximizes u_i on

$$\{\xi_i'' \mid \exists (x_i', \xi_i') \in T_i,\ q_i \cdot \xi_i'' \leq p \cdot x_i + (q_i \cdot \xi_i' - p \cdot x_i')\},$$

(ii) $\sum_{i=1}^m x_i$ maximizes $p \cdot y$ on Y.

In the market equilibrium with characteristics prices the consumer may (at least formally) buy *both* commodities and characteristics at the given prices, and in optimum the best bundles in the consumption set are chosen. Note that due to the feasibility constraint, no actual transfers of characteristics take place; the characteristics prices are only used as constraints to the individual optimization of the consumer.

Theorem 1 *Let $\mathcal{E}$ be an economy satisfying Assumption 1. If (x, ξ, p, q) is a market equilibrium with charcteristics prices, then the allocation (x_i, ξ_i) is Pareto optimal. Conversely, let (x, ξ) be a Pareto optimal allocation in $\mathcal{E}$, where $\xi_i \in \mathbf{R}_{++}^L$ for all i. Then there is price system p on commodities and a system of individual characteristics prices $q = (q_i)_{i=1}^m$ such that (x, ξ, p, q) is a market equilibrium with characteristics prices.*

The proof of Theorem 1, which uses the standard method of embedding the given economy in an ordinary economy with a larger commodity space, is given in the appendix.

The result of Theorem 1 provides the basis for using characteristics or shadow prices in welfare considerations since changes in overall welfare may be evaluated by computing the aggregate value of the characteristics consumption. Following up on the discussion in Section 2, it can be noticed, however, that such a welfare measure would use individual shadow prices, which might very well be different for different individuals and the basic problem is therefore to find conditions on the economy making the characteristics prices q_{ij}, $i = 1, \ldots, m$, of any characteristic j, identical.

4 Equality of individual characteristics prices: Some examples

In the present section, we demonstrate by examples that equality of characteristics prices does not prevail in general, even in simple two person economies. The examples will, however, help to focus on the exact conditions under which equality indeed occurs.

In the first example, consumers have identical household technologies but differ in preferences.

Example 1. Suppose that the economy $\mathcal{E}$ has two consumers and one (marketed) commodity x, (i.e. $l = 1$). Consumers transform arbitrary amounts of this commodity into bundles of two different characteristics (ξ_1, ξ_2) (i.e. $L = 2$) using a common household technology T. The household technology is assumed to be given by $(x, (\xi_1, \xi_2)) \in T$ if

$$(\xi_1^2 + \xi_2^2)^{1/2} \leq x.$$

Thus, the characteristics are made under conditions of joint production and constant returns to scale.Consumers have the utility functions

$$\begin{aligned} u_1(\xi_{11}, \xi_{12}) &= \xi_{11}\xi_{12}^4 \\ u_2(\xi_{21}, \xi_{22}) &= \xi_{21}^4\xi_{22}. \end{aligned}$$

Finally, there is an initial endowment of 1 unit of the commodity but no commodity production, i.e. $Y = \{y \in \mathbf{R} \mid y \leq 1\}$.

We obtain all Pareto optimal allocations (x, ξ) by dividing the initial endowment between consumers and letting them produce characteristics from this endowment such as to maximize their utility. Thus, let $x_1 \in [0, 1]$; then (ξ_{11}, ξ_{12}) and (ξ_{21}, ξ_{22}) satisfy

$$\xi_{11}^2 + \xi_{12}^2 = x_1^2, \quad \xi_{21}^2 + \xi_{22}^2 = (1 - x_1)^2.$$

Adding the individual maximization conditions, we get that

$$(\xi_{11}, \xi_{12}) = \left(\frac{x_1}{\sqrt{5}}, \frac{2x_1}{\sqrt{5}}\right), \quad (\xi_{21}, \xi_{22}) = \left(\frac{2(1 - x_1)}{\sqrt{5}}, \frac{1 - x_1}{\sqrt{5}}\right).$$

Therefore the marginal rate of substitution between characteristics for consumer 1 is given by

$$\mathrm{MRS}_1 = \frac{\partial u_1}{\partial \xi_1} / \frac{\partial u_1}{\partial \xi_2} = \frac{1}{4}\frac{\xi_{12}}{\xi_{11}} = \frac{1}{2},$$

and similarly, the marginal rate of substitution for consumer 2 is $\mathrm{MRS}_2 = 2$. As such, the marginal rates of substition are independent of the parameter x_1. Moreover, MRS_1 and MRS_2 remain the same in all the Pareto optimal allocations (except for $x_1 = 0$ or $x_1 = 1$, where any individual price vector is a support for the preferred set of the individual getting nothing). We conclude that the marginal rates of substitutions differ for all Pareto optimal allocations.

Indeed, it easy to obtain more general results stating that if all consumers behave according to standard assumptions (i.e. they all have the same household technology with constant returns to scale as well as homothetic preferences which are sufficiently different) then there are no equlibria with identical characteristics prices. In fact, as the following example will show, even with identical household technology and identical preferences we will not automatically obtain identical characteristics prices if the assumption of constant returns to scale is abandoned.

Example 2: Suppose that the common household technology T is such that $(x, \xi) \in T$ if

$$(t\xi_1^2 + \xi_2^2)^{1/2} \leq t, \ t \leq x,$$

and that both consumers have the utility function u with

$$u(\xi_1, \xi_2) = \xi_1 \xi_2.$$

As before, we find all Pareto optimal allocations by choosing $x_1 \in [0, 1]$ and maximizing u under each of the constraints

$$x_1\xi_{11}^2 + \xi_{12}^2 = x_1^2, \quad (1 - x_1)\xi_{21}^2 + \xi_{22}^2 = (1 - x_1)^2.$$

This yields characteristics bundles

$$(\xi_{11}, \xi_{12}) = \left(\sqrt{\frac{x_1}{2}}, \frac{x_1}{\sqrt{2}}\right), \quad (\xi_{21}, \xi_{22}) = \left(\sqrt{\frac{(1 - x_1)}{2}}, \frac{1 - x_1}{\sqrt{2}}\right).$$

Once more we have inequality of marginal rates of substitution, which with the particular utility function are $\mathrm{MRS}_1 = \sqrt{x_1}$ and $\mathrm{MRS}_2 = \sqrt{1 - x_1}$, respectively, differing except in the case $x_1 = (1 - x_1)$ where the identical consumers are treated equally.

Having shown above that equality of individual characteristics prices will fail in many situations, we shall subsequently concentrate on finding conditions under which equality does actually obtain. Basically, there are four cases;

(1) All consumers have different household technologies *and* different utility functions. In this case equality of marginal rates of substitution for characteristics would obtain only exceptionally.

(2) All consumers have identical household technologies *and* identical utility functions. This means that there is essentially only one consumer, so the problem of equality becomes trivial. It may be remarked, however, that even here the situation is not entirely trivial as shown by Example 2, where inequality of individual characteristics prices may occur even if both household technologies and preferences are identical across individuals due to non-constant returns to scale in technology.

(3) All consumers have a common household technology but arbitrary individual utility functions. This case will be considered in Section 5

(4) All consumers have a common utility function but arbitrary individual household technologies. This case is treated in Section 6.

5 Equality of individual characteristics prices:The case of a common household technology

In this section, we consider the case where every individual has access to the same technology for transforming commodities into characteristics. Dealing with health-related characteristics, this means that the household technology contains all procedures by which services and goods available in the market can be used to improve individual health in one or several of its aspects. Needless to say that this is a somewhat restrictive assumption, since some methods of obtaining health, both by treatment and by the way of conducting your life, may not be known, or indeed possible, for everybody. Therefore, we shall also examine the general case of differing household technologies in the next section.

As it appeared in the previous section, different individual characteristics prices may be caused by a transformation curve in characteristics space where the slope depends on the chosen output bundle. In this case, assigning suitable preferences over characteristics to individuals, it is always possible find market equilibria for which one individual has a marginal valuation of some characteristic which differs from that of some other individual. If the transformation curve is a straight line, however, this cannot occur. Indeed, in optimum the slope of a linear transformation curve will define characteristics prices which are obviously the same for all. In the following we shall hence consider a general form of such a linear-transformation-curve condition.

A household technology $T \subset \mathbf{R}^l_+ \times \mathbf{R}^L_+$ is said to be *separable* if $T = \sum_{j=1}^L T^j$ with

$$T^j \subset \mathbf{R}^l_+ \times \{\xi \in \mathbf{R}^L_+ \mid \xi_k = 0,\ k \neq j\}$$

for $j = 1, \ldots, L$. In other words, a technology is separable if each of the characteristics are produced in a separate technology; producing a characteristics bundle $\xi = (\xi_1, \ldots, \xi_L)$ means that ξ_1 is produced in the technology T^1 having the first characteristic as output, ξ_2 similarly in the specific technology T^2 for producing characteristics number 2, etc. In such a technology, what can be produced with a given input of commodities is obviously the convex hull of each of the productions obtainable by putting everything into the technology T^j producing the jth characteristic, for $j = 1, \ldots, L$. This clearly means that transformation surfaces are hyperplanes in $\mathbf{R}^L_+$, thus generalizing the condition to be inferred from the simple cases in Section 4.

Intuitively, separability if the household technology means that what the individual does in order to produce some characteristic j has no side-effects or spin-offs on the production of any other characteristic. Thinking of characteristics as representing enjoyment of health in its various forms, this is not a particularly attractive property – many activities which enhance one kind of health will typically be beneficient also for other aspects of the individual's health, and on the contrary some efforts to promote a particular form of health may have detrimental effects on other aspects of health. It seems rather unlikely that health characteristics, chosen in such a way as to be intuitively meaningful and observable, should be mutually independent in the sense that you can have one aspect of health with no repercussions on other aspects.

However, relevant or not, it is a fact that the separability assumption is quite crucial if characteristics prices are to be identical in equilibrium, as demonstrated by the following result.

Theorem 2 *Let $T \subset \mathbf{R}^l_+ \times \mathbf{R}^L_+$ be a household technology satisfying constant returns to scale, and let $\mathbf{E}_T$ be the class of economies satisfying Assumption 1 such that all consumers have the household technology T. Then the following conditions are equivalent:*

(i) for all economies $\mathcal{E} \in \mathbf{E}_T$, all equilibria (x, ξ, p, q) of $\mathcal{E}$ with $\xi_i \neq 0$ for all i satisfy $q_1 = \cdots = q_m$,

(ii) the household technology T is separable.

The proof of Theorem 2 is given in the appendix. The result states that if we are to reckon with equality of individual characteristics prices in the sense that it is not achieved due to some coincidence of individual preferences, then the method of producing characteristics from commodities (i.e. the household technology) must satisfy separability. If not, there will be some constellation of individual utility functions such that individuals value small changes in health of a particular aspect differently, measured either in terms of other aspects of health or in monetary terms. If this occurs, then the weight of this aspect of health is ambiguous, and there is no simple answer to the question of how to set the weight of this particular aspect of health in a cost-effectiveness analysis (as discussed in Section 2).

6 Equality of individual characteristics prices:The case of common utility functions

We now turn to the case where individuals no longer have access to the same household technology. In our interpretation, this means that the same treatement may effect people differently, or that there may be medical treatments which are not open to all individuals, either because they are not admitted, or simply because they are not aware of their existence. As such, the assumption of individual household technologies may seem more natural than the assumption of a common household technology, as in the previous section.

As mentioned earlier, if both household technologies and utility functions differ, then equality of individual characteristics prices would obtain only as an exceptional case. Therefore, we examine the situation where all individuals have the same utility function.

Intuitively, identical preferences imply consensus among all individuals on the general way of assessing alternative characteristics bundles. On the other hand, due the difference in household technology, consumers will typically end up with different characteristics bundles and this, in turn, will typically give rise to different marginal rates of substitution even though the underlying utility function is the same. As such, the interpretation follows the standard case of inequality in access to health – if some individuals cannot use the health improving methods that are available to others, the result will be an inferior state of health in certain respects. As a consequence, the marginal utility of an improvement in the relevant aspects of health would typically be greater for those people than for the rest of the population.

The case where such differences do *not* occur, making aggregation possible in a meaningful way, almost suggests itself; this is the case where marginal rates of substitution are independent of the bundle chosen, so that indifference surfaces are hyperplanes. As it can be seen from the statement of the result, it may be considered as a dual of Theorem 2, since the separability condition on preferences takes the place of the previous separability condition for household technologies. It is assumed that preferences are homothetic and thereby can be described by a utility function that is homogeneous (of degree one); in this way we exclude the possibility that differences in marginal rates of substitution might arise only due to differences in wealth.

THEOREM 3 *Let u be a given homogeneous utility function on characteristics, let $\mathcal{T}$ be the set of all household technologies which satisfy constant returns to scale, and let $\mathbf{E}_u$ be the set of all economies satisfying Assumption 1 such that each individual i has utility function u and has household technology $T_i \in \mathcal{T}$. Then the following are equivalent:*

(i) for all economies $\mathcal{E} \in \mathbf{E}_u$, all equilibria (x, ξ, p, q) of $\mathcal{E}$ with $\xi_i \neq 0$ for all i satisfy $q_1 = \cdots = q_m$,

(ii) the common utility function is separable, that is there are functions $u_j : \mathbf{R}_+ \to \mathbf{R}$, $j = 1, \ldots, L$, such that

$$u(\xi_1, \ldots, \xi_L) = \sum_{j=1}^{L} u_j(\xi_j)$$

for all characteristics bundles $(\xi_1, \ldots, \xi_L)$.

The proof of Theorem 3 is given in the appendix. The result shows that if we are to expect that individual characteristics prices will be identical, then the utility function must be (additively) separable. The intuition behind this result is as follows: Even though people evaluate health profiles in the same way, they may come up with very different bundles due to their differing access to health; when bundles differ, it might well happen that the marginal rates of substitutions (the slopes of the indifference curves in the two-dimensional case) are different, and that would give us a contradiction. Therefore, the indifference curves must have the same slope everywhere, which means that utility functions must be separable.

Separability, in its turn, implies preference independence of the various health characteristics in the sense that if a given combination of some characteristics is preferred to another combination of the same characteristics, given identical assignment in the remaining characteristics, then it will be preferred no matter what this assignment in the remaining characteristics looks like. Separability is usually considered a very strong assumption on preferences.

Separability or independence assumptions on individual preferences is usually assumed when methods for assigning values to health status indices are proposed. For instance, Sintonen (1981) determines linear weights assigned to each of the 11 basic aspects of health, and the final index is then found as the weighted average. Also in QALY measurement, separability enters as an essential assumption; the time tradeoff methodology assumes (multiplicative) separability in life expectation and quality-of-life, and the standard gamble relies on von Neumann-Morgenstern utility representation of lotteries over life and death, thereby implicitly assuming preference independence (cf. Dolan, 2000). Thus, the authors concerned with finding a logical basis for health status measurements have typically relied on separability or independence assumptions; what is added by our result is that this assumption is indeed necessary.

7 Final remarks

Thoughout this paper, we have been concerned with welfare economic foundations of health status measures. As argued in Section 2, if health status measures shall be meaningful from a welfare economic point of view then the marginal rate of substitution between any two aspects of health must be the same for all individuals. Since a similar property holds due to the effects of the market for commodities, it might very well be the case that the market, although indirectly, could produce this equality also for (health) characteristics.

It turned out that although we cannot hope for such results holding in broad generality, they do hold under separability assumptions on either the technology for transforming goods into health characteristics or on the utility functions defined on such characteristics. These separability conditions are usually considered as restrictive; separability in technology means that each health characteristic is produced in a way which is independent of the quantity produced of all the other characteristics, and separability of utility functions means that marginal rates of substitution between any two characteristics are independent of the level of all the remaining characteristics. Although these assumptions are restrictive, some of the very succesful fields of applied economics (such as the Heckscher-Ohlin model of internation trade with the celebrated factor price equalization theorem) relies on exactly the same kind of assumptions. Thus, theorem 2 and 3 may be considered as a welfare theoretical basis for current measurement of health status by indices such as the QALY.

If simple methods of aggregation fail, there is of course a need for other methods which do not use *ad hoc* aggregation, and the use of health status indices may indeed be justified by the fact that so far they are the best avaiable. But this does not mean that evaluating medical treatments or new pharmaceutical products cannot be performed without aggregating health characteristics into a single number to be put in the denominator of a cost-effectiveness ratio. Indeed, the approach outlined in Section 2 shows that even the cost-effectiveness ratio itself is a means of avoiding an aggregation (namely of characteristics related to health and commodities), and in this perspective, it should not be beyond reach to avoid also some of the aggregations which are inherent in the approach to outcome measurement using health status indices. This, however, is beyond the scope of the present paper.

8 References

Baldwin,S., C.Godfrey, and C.Propper (eds.) (1990), Quality of Life, Perspectives and Policies, Routledge, London and New York.

Bleichrodt,H., P.Wakker and M.Johannesson (1997), Characterizing QALYs by risk neutrality, Journal of Risk and Uncertainty 15, 107 – 114.

Bleichrodt,H. and J.Quiggin (1997), Characterizing QALYs under a general rank dependent utility model, Journal of Risk and Uncertainty 15, 151 – 165.

Broome,J. (1993), QALYs, Journal of Public Economy 50, 149-167.

Brouwer, W.B.F. and M.A. Koopmanschap (2000), On the economic foundations of CEA. Ladies and gentlemen, take your positions!, Journal of Health Economics 19, 439-459.

Culyer,A.J. (1990), Commodities, characteristics ofcommoditites, characteristics of people, utilities, and the quality of life, in Baldwin, Godfrey, and Popper (1990).

Culyer, A.J. and J.P. Newhouse (eds.) (2000), Handbook of Health Economics, Elsevier.

Dolan, P. (2000), The measurement of health-related quality of life, Chapter 32 in Culyer and Newhouse (2000).

Garber, A.M. (2000), Advances in CE analysis, Chapter 4 in Culyer and Newhouse (2000).

Gold,M.R., J.E.Siegel, L.B.Russell, M.C.Weinstein (eds.) (1996), Cost-effectiveness in health and medicine, Oxford University Press, New York.

Grossman,M. (1972), On the Concept of Health Capital and the Demand for Health, Journal of Political Economy 70, 223 – 255.

Laffont, J.J. (1988), Fundamentals of public economics, MIT Press.

Lancaster,K. (1971), Consumer Demand, A New Approach, New York – London.

Lessourne, J. (1975), Cost-benefit analysis and economic theory, North-Holland.

Loomes,G. and L.McKenzie (1989), The Use of QALYs in Health Care Decision Making, Social Science and Medicine 28, 299 - 308.

Sintonen,H. (1981), An approach to measuring and valuing health states, Social Science and Medicine, 15C.

Torrance,G.W. (1986), Measurement of health state utilities for economic appraisal, Journal of Health Economics 12, 39 – 53.

Torrance,G.W., and D.Feeny (1989), Utilitites and Quality-Adjusted Life Years, International Journal of Technology Assessment in Health Care 5, 559 – 575.

Ware,J.E. (1995), The status of health assessment 1994, Annual Rev. Public Health 16, 327 – 354.

Williams,A. (1985), The value of QALYs, Health and Social Service Journal (July 18).

9 Appendix: Proofs of theorems

Proof of Theorem 1: The proof that the allocation (x, ξ) associated with an equilibrium (x, ξ, p, q) is Pareto optimal (that is Main Theorem 1 of economic welfare theory holds) is standard, and we leave it to the reader. For the converse, assume that (x, ξ) is Pareto optimal.

For each consumer i, define the set

$$P_i^0 = \{\xi_i' \in \mathbf{R}_+^L \mid u_i(\xi_i') \geq u_i(\xi_i)\};$$

P_i is closed and convex since u_i is continuous and quasi-concave; we embed P_i in $\mathbf{R}^{l+mL}$ putting 0 in all places except in the ith of the m segments of L coordinates. Denote the resulting set by $\tilde{P}_i$.

For each i, let $\tilde{T}_i$ be the set T_i embedded in $\mathbf{R}^{l+mL}$ by putting 0 in all coordinates except the first l, where the sign is reversed, and the ith segment of L coordinates. Again, $\tilde{T}_i$ is closed and convex. Finally, the closed and convex set $Y \subset \mathbf{R}^l$ is embedded in $\mathbf{R}^{l+mL}$ as $\tilde{Y}$ by adding 0s in all but the first l coordinates.

Now consider the set

$$Z = \sum_{i=1}^{m} \tilde{P}_i - \sum_{i=1}^{n} \tilde{T}_i - \tilde{Y};$$

We have that Z is closed and convex, and that it contains the zero vector, which has the representation

$$0 = \sum_{i=1}^{n} \tilde{\xi}_i - \sum_{i=1}^{n} \widetilde{(x_i, \xi_i)} - \tilde{y}$$

for some $y \in Y$ (here $\tilde{a}$ denotes the image of a by the $\tilde{\ }$-mapping). We claim that $Z \cap \mathrm{Neg} = \emptyset$, where Neg is the cone of negative vectors in $\mathbf{R}^{l+mL}$. Suppose to the contrary that $u \in Z \cap \mathrm{Neg}$, so that

$$u = \sum_{i=1}^{n} \tilde{\xi}_i' - \sum_{i=1}^{n} \widetilde{(x_i', \xi_i')} - \tilde{y}'$$

for some $y' \in Y$; but this means that the allocation (x', ξ') is at least as good as (x_i, ξ_i) and that there is a feasible allocation (x_i'', ξ_i) with $x_i'' > x_i'$ for all i. By the monotonicity properties of T_i and u_i, $i = 1, \ldots, m$, there is then a feasible allocation (x_i'', ξ_i'') with $u_i(\xi_i'') \geq u_i(\xi_i)$, contradicting Pareto optimality.

Now, by separation of the convex sets Z and Neg, there exists a non-negative vector $(p, q) = (p, (q_i)_{i=1}^m) \in \mathbf{R}^{l+mL}$ such that the scalar product $(p, q) \cdot z$ for $z \in Z$ is minimized at $z = 0$. But this means that (a) y maximizes $p \cdot y$ on Y, which is condition (ii) of market equilibrium with characteristics prices; furthermore it means that (b) ξ_i minimizes $q_i \cdot \xi_i'$ on P_i, which, by the interiority condition, implies that $q \cdot \xi_i'' > q \cdot \xi_i$ for all ξ_i'' with $u_i(\xi_i'') > u(\xi_i)$, and that (c) (x_i, ξ_i) maximizes $q_i \cdot \xi_i' - p \cdot x_i'$ on T_i.

To check equilibrium condition (i), let $i \in \{1, \dots, m\}$ and assume now that ξ_i'' satisfies the inequality

$$q_i \cdot \xi_i'' \leq p \cdot x_i + \max_{(x_i', \xi_i') \in T_i} [q_i \cdot \xi_i' - p' \cdot x_i'].$$

Then from (c) we get that $q_i \cdot \xi_i'' \leq p \cdot x_i + [q_i \cdot \xi_i - p \cdot x_i] = q_i \cdot \xi_i$, and (b) implies that $u_i(\xi_i) \geq u_i(\xi_i'')$, which is (i). □

Proof of Theorem 2: (ii) ⇒ (i): Let $\mathcal{E} \in \mathbf{E}_T$, and let (x, ξ, p, q) be an equilibrium. Then by the condition (i) of an equilibrium, for each i the characteristics bundle ξ_i is maximal for u_i on the set of all

$$\{\xi_i'' \mid q_i \cdot \xi_i'' \leq p \cdot x_i + \max_{(x_i', \xi_i') \in T_i} [q_i \cdot \xi_i' - p \cdot x_i']\} = \{\xi_i'' \mid q_i \cdot \xi_i'' \leq q_i \cdot \xi_i\};$$

clearly, this set contains

$$T_i(p) = \{\xi_i' \mid \exists x_i' : \, p \cdot x_i' \leq p \cdot x_i, \, (x_i', \xi_i') \in T_i\},$$

and ξ_i belongs to the boundary of $T_i(p)$. Now, by constant returns to scale of T, we have that $T_i(p) = \lambda_i T(p)$, where $\lambda_i = p \cdot x_i$, and using (ii) we get that

$$T(p) = \lambda^{-1} \{\xi_i'' \mid q_i \cdot \xi_i'' \leq q_i \cdot \xi_i\}.$$

Since the left hand side is independent of i, so is the right hand side, and we conclude that $q_1 = \cdots = q_m$.

(i) ⇒ (ii): Let $C = \{c^1, \dots, c^i, \dots\} \subset \mathbf{R}^L_{++}$ be a countable dense subset of $\mathbf{R}^L_+$, and let $\hat{p} \in \mathbf{R}^l_{++}$ be arbitrary. For each natural number m, we define an economy $\mathcal{E}_m \in \mathbf{E}_T$ with m consumers $i = 0, 1, \dots, m-1$ as follows: Consumer 0 has preferences such that $u_i(\xi') \geq u_i(\xi)$ if and only if $\sum_{j=1}^L \xi_j' \geq \sum_{j=1}^L \xi_j$. For $i = 1, \dots, m-1$, let the utility function of consumer i be given by

$$u_i(\xi) = \min_{j=1,\dots,L} \frac{\xi_j}{c_j^i}.$$

The aggregate availability set $Y \subset R^l$ is defined by

$$Y = \{y \in \mathbf{R}^l \mid p \cdot y \leq 0, y_1 \leq 1\},$$

(so that commodity 1 is available in the magnitude 1 and can be used as input, whereas the other commodities can only be obtained as output).

Let (x, ξ, p, q) be an extended equilibrium in $\mathcal{E}_m$. By assumption, $q_0 = \cdots = q_{m-1} = \bar{q}$. We claim that $\sum_{i=0}^{m-1} \xi_i > 0$. Suppose not, then there is $j \in \{1, \dots, L\}$ such that $\xi_{ij} = 0$ for all i and consequently $u_i(\xi_i) = 0$ for $i \geq 1$. If $\xi_i \neq 0$, then also $x_i \neq 0$ (here we use relevance, cf. Assumption 1(ii)), and then by monotonicity of T (also in Assumption 1(ii)) there is ξ_i' with $u_i(\xi_i') > u_i(\xi_0)$ with $(x_0 + x_i, \xi_0') \in T$, contradicting the Pareto optimality of (x, ξ) (if consumer i gets utility 0 anyway then her commodities could just as well be given to consumer 0, who would become better off). This proves our claim.

By Assumption 1(ii) (relevance), we now conclude that $\sum_{i=1}^m x_i > 0$, and it follows from equilibrium condition (ii') that $p = \hat{p}$. Arguing as above, we have that ξ_i maximizes $\bar{q} \cdot \xi$ on

$$\{\xi \mid \exists x : \hat{p} \cdot x \leq \hat{p} \cdot x_i, (x, \xi) \in T_i\},$$

and by constant returns to scale, we have that $\lambda_i^{-1}\xi_i$ maximizes $\bar{q} \cdot \xi$ on $T(\hat{p})$, where $\lambda_i = \hat{p} \cdot x_i$ for each i. We conclude that $\mathrm{conv}(\{\lambda_i^{-1}\xi_i \mid i = 1, \ldots, m-1\})$ belongs to the intersection of $\mathrm{bd}\, T(\hat{p})$ with a hyperplane $\{\xi \mid \bar{q} \cdot \xi = M\}$, where $M = \lambda_i^{-1}\bar{q} \cdot \xi_i$ is independent of i.

Next, we notice that $\bar{q}_j > 0$ for all h since otherwise consumer 0 would not satisfy the individual optimality constraint (i') at ξ_0. But this means that for $i \geq 1$,

$$\lambda_i^{-1}\xi_i = \frac{M}{\bar{q} \cdot c_i} c_i,$$

so that for m large enough, the set $\mathrm{conv}(\{\lambda_i^{-1}\xi_i \mid i = 1, \ldots, m-1\})$ gets as close as desired to $\{\xi \in \mathbf{R}_+^L \mid \bar{q} \cdot \xi = M\}$. But then

$$T(\hat{p}) = \{\xi \in \mathbf{R}_+^L \mid \bar{q} \leq M\},$$

and since $\hat{p}$ was arbitrary, we have shown that for all $p \in \mathbf{R}_{++}^l$, the set $T(p)$ is the intersection with $\mathbf{R}_+^L$ of a half-space in $\mathbf{R}^L$. The rest of the proof consists in showing that this latter property implies that T is separable.

For $j = 1, \ldots, L$, define T^j by

$$T^j = T \cap \{(x, \xi) \in \mathbf{R}_+^l \times \mathbf{R}_+^L \mid \xi_k = 0, k \neq j\}.$$

Then T^j is a convex cone contained in T, each j, and $\sum_{j=1}^L T^j \subset T$.

Assume that $(x, \xi) \in T$ but $(x, \xi) \notin \sum_{j=1}^L T^j$. Then for every array $(x^1, \ldots, x^L) \in (\mathbf{R}^l)^L$ such that $(x^j, e^j) \in T^j$ for each j (where e^j is the jth unit vector in $\mathbf{R}^L$), we have that $x \neq \sum_{j=1}^L \xi_j x^j$. The set

$$C = \left\{ \sum_{j=1}^L \xi_j x^j \mid (x^j, e^j) \in T^j, \, j = 1, \ldots, L \right\}$$

is closed and convex (by convexity of each of the sets T^j), and

$$\{x' \in \mathbf{R}^l \mid x' \leq x\} \cap C = \emptyset$$

(by the free disposal property of household technologies), so by separation of convex sets, there is $p \in \mathbf{R}_{++}^l$ with

$$\begin{aligned} p \cdot x &= 1, \\ p \cdot x' &> 1 \text{ for } x' \in C. \end{aligned}$$

However, by the assumptions of the Theorem, we know that there are $(\hat{x}^j, \hat{\xi}^j) \in T^j$ with $p \cdot \hat{x}^j = 1$, $j = 1, \ldots, L$, such that

$$\xi = \sum_{j=1}^{L} \lambda_j \hat{\xi}^j$$

for some $\lambda_1, \ldots, \lambda_L \geq 0$ with $\sum_{j=1}^{L} \lambda_j = 1$. Clearly, for each j we have that $\lambda_j \hat{\xi}^j_j = \xi_j$, meaning that $(\lambda_j \hat{x}^j, \xi_j e^j) \in T_i$, and

$$\sum_{j=1}^{L} \lambda_j \hat{x}^j \in C.$$

However, since $p \cdot \hat{x}^j = 1$ for each j, we get that

$$p \cdot \left(\sum_{j=1}^{L} \lambda_j \hat{x}^j \right) = \sum_{j=1}^{L} \lambda_j (p \cdot \hat{x}^j) = 1,$$

contradicting that $p \cdot x' > 1$ for each $x' \in C$. □

Proof of Theorem 3: The proof of the implication $(ii) \Rightarrow (i)$ follows the same line of reasoning as in Theorem 2 and is left to the reader.

$(i) \Rightarrow (ii)$: Consider the convex set $C(u) = \{\xi \mid u(\xi) \geq 1\}$; we say that $\pi \in \mathbf{R}^L_+$ is a support of $C(u)$ if $\pi \cdot \xi' \geq \pi \cdot \xi$ for all $\xi' \in C(u)$.

Let $\{c_1, c_2, \ldots\}$ be a countable set of points in $\mathbf{R}^L_{++}$ which are dense $\{c \in \mathbf{R}^L_+ \mid c_1 + \cdots + c_L = 1\}$. For each natural number m, consider the economy $\mathcal{E}_m$ with m consumers, where each consumer i has utility function u and household technology

$$T_i = \{(x_i, \xi_i) \mid \xi_i \leq \|x_i\| c_i\},$$

$i = 1, \ldots, m$, and where $Y = \{x \mid x \leq (1, \ldots, 1)\}$.

For each m, let (x, ξ, p, q) be a market equilibrium of $\mathcal{E}_m$ which by assumption satisfies $q_1 = \cdots = q_m = q^0$. Then $u(\xi_i) = u(\|x_i\| c_i)$ for each i, and without loss of generality we may assume that $\xi_i = \|x_i\| c_i$, each i. From this it follows that $C(u)$ has the same support q^0 at all points where rays given by c_i intersect $C(u)$. As m tends to infinity, we conclude that $C(u)$ is the intersection of $\mathbf{R}^L_+$ with a halfspace, so that u is indeed affine. □