Health insurance, competition and auction

David Bardey[*], Crest-Lei,

Pierre-Henri Morand, Crese, University of Besançon.

July 14, 2002

Abstract

The optimal organization of the health insurance sector is a very controversial question. In his famous article, Diamond (1992) suggested to regulate competition in health insurance markets through the auction mechanisms. Public authorities create groups of policy holders which are allocated among insurers according to an auction procedure. In this article, we complete Diamond's analysis by determining the optimal auction according to the health insurance sector's characteristics. Our findings suggest that split award auctions of incentive contracts must be used to introduce an efficient ex ante competition between insurers.

Keywords : health risk, collective insurance, incentive contracts, split award auction.

Jel Classification : L52-I18-D44

[*]Correspondence to : David Bardey, Crest-LEI / ENPC, 28 rue des Saints Pères, 75007 Paris, France. E-mail : bardey@ensae.fr

The optimal organization of the health insurance sector constitutes an old and very controversial discussion among health economists. The individual health insurance markets possess several specific failures such as adverse selection, risk selection and inequity. Adverse selection comes from the fact that the policy holders may possess private information about their own risk which can in certain cases destabilize the health insurance markets.[1] Health insurance markets in their pure form can appear unfair when policy holders pay insurance premiums according to their individual risk if a main part of this risk is independent from the policy holders' behaviors. Besides, actuarial tariffs may exclude some people from the health insurance markets if premiums represent too big a part of their income[2].

Risk selection strategies may be used by insurers when they find that these strategies are more profitable than the refinement of the actuarial tariffs. Pauly (1984) added that the profitability of the strategies of risk selection is higher when a policy regulation prevents insurers from establishing actuarial premiums. Finally, some people may sometimes choose not to suscribe an insurance contract because some free care is available for the uninsured.[3]

All the failures mentioned are specific to the individual health insurance markets. Then, two solutions are usually suggested to manage the health risk more efficiently.

The first one, a bit radical, is to set up a public monopoly in charge of managing the health risk of the entire population. Health economists often explain that the health insurance sector may be characterized by increasing returns of scale which constitute an important argument in favor of the public system.[4] Besides, the public

[1] See Rothchild and Stiglitz (1976) for a theoretical model of this phenomena and Cutler and Reber (1998) for an empirical assessment in the United States.

[2] This problem is treated by Couffinhal, Henriet and Rochet (2000).

[3] This last problem can be reduce thanks to vouchers for insurance purchase (see Pauly and al., 1991).

[4] For example Blair and al. (1975).

organization of health insurance allows to operate two kinds of redistribution among policy holders: redistribution of risks and redistribution of income. Nevertheless, all the arguments in favor of public systems must be weighed up with the lack of incentives which is a consequence of the monopoly structure. A quotation by Pauly (2000) tackles this dilemma: "*The problem with monopoly*[5] *is the same as in the other industries*". Efficiency gains due to returns of scale may be dissipated as "*administrative slack*".

The second one is the collective insurance where policy holders belong to large groups when they subscribe their health insurance contracts. Thanks to the separation between premiums paid and individual risks, this kind of management of the health risk permits to implement cross-subsidization between the policy holders. So, risk selection and adverse selection are eliminated for the same reasons and equity considerations are improved. The main problem of collective health insurance contracts is that they are usually subscribed through the employer. Diamond (1992) explained that this organization reduces the labor mobility, which implies that job market allocations may be inefficient.[6]

This author suggested a very tempting solution to this problem. He propounded that public authorities create groups of policy holders which are assigned to insurers according to an auction procedure.[7] This attractive solution implies an ex ante competition between insurers and therefore permits to keep the advantages of the monopoly structure and, at the same time, to introduce competitive pressure. Contrary to the individual health insurance market, this type of regulation orientates the

[5] Monopoly of health insurance.

[6] For example, a policy holder who fall ill and who is characterized by a chronic desease will never change his job because of the premium risk.

[7] In Diamond's article, groups of policy holders are created at the Federal level. We shall explain in the conclusion that in the United States' case, this solution may be not optimal if insurers can participate in auction procedures in several states.

competitive pressure in the direction of the search for more efficiency. However, Diamond did not specify the modalities of the auction procedure that must be used to allocate efficiently groups of policy holders among insurers. Now, auction procedures are very diversified[8] and the optimal procedure crucially depends on the industries' characteristics. The purpose of this article is to determine the optimal auction procedure according to the characteristics of the health insurance sector.

In the first section, we shall describe the main characteristics of the health insurance sector. This description will enable us to focus on auctions of incentive contracts. Besides, we shall also explain that the presence of several groups of policy holders necessitates to consider multi-unit auctions. In the second section, we shall propound a theoretical model which deals with split award auctions of incentive contracts. We shall conclude in the third section.

CHARACTERISTICS OF THE HEALTH INSURANCE SECTOR AND AUCTION

In this section, we deal with three points. In the first paragraph, we describe the institutional environment in which an ex ante competition can be implemented. In the second paragraph, we tackle with the cost function of the insurers. We conclude that the auctionning of incentive contracts must be considered. In the third paragraph, we deal with returns of scale in the health insurance sector. Then, we explain that the nature of the optimal auction procedure depends on economies of scale.

Institutional environment and ex ante competition

Before determining the optimal auction mechanism adapted to the health insurance sector, we must describe the institutional environment.

[8] Cf. Klemperer (1999).

First, the regulator must divide the population to create several groups of policy holders. These groups can indifferently be composed by homogeneous policy holders or by people who are characterized by different levels of risk. This organization allows to redistribute risks among policy holders. Besides, insurance premiums are paid to the regulator and not directly to the insurers. So, income redistribution through health insurance can also take place. This property of our organization is important because health insurance is often mentioned as an efficient means of redistribution, complementary to income taxation.[9]

As Diamond suggests, we consider that the formation of groups must be decided by the regulator. This implies restrictions on individual choice because policy holders cannot choose their insurers. We also consider that the regulator imposes a norm on collective health insurance contracts in order to prevent insurers from distorting competition. It seems more fair to standardize the quality of the insurers' supplies when policy holders have not the choice of their insurers. This standardization of the health insurance contracts allows us to restrict our attention to auctions which do not depend on the quality of the contracts.[10] The standardization must define the health insurance coverage rate but also the pannier of care which is covered. Diamond also mentioned that competition on quality is very difficult to implement because of the complexity of the comparison.

To end with this point, we note that this mode of regulation is compatible with a complementary health insurance market if a copayment rate is specified through the standardization of contracts.

[9] Rochet (1991), Henriet-Rochet (1999) and Jack (1999).

[10] The determination of the optimal auction in a multidimensionnal context (price, quality) is very difficult when quality is not perfectly observable.

Cost function of the insurers and auctioning of incentive contracts

In a context of collective insurance, the cost function of the insurers depends mainly on three factors.

The first one is the aggregated level of risk of the policy holders who compose the group. The groups of policy holders are composed by public authorities. As we have already noted, the health risk of policy holders who compose a group can be different or identical. Public authorities can also create groups which are characterized by the same aggregated level of risk or by different levels. We shall assume in the next section that the regulator, as the insurers, can observe the main characteristics of the policy holders who compose a group, that only implies that aggregated levels of risk of the groups are observable for both parties (this is a realistic assumption for collective contracts).

The second factor is the level of insurers' productivity for managing health risks. All the insurers have not the same productivity, so one of the objectives which is assigned to the auction procedure is to select the most efficient ones. In the model of auction that we suggest, we shall assume that the productivity of insurers is not observable. In the insurance terminology, the productivity of the insurance companies is measured with loading factors.

Thirdly, the health risk is specific because the cost function of the insurers depends on the price of the care and on their degree of intervention in the health care sector. Schokkaert and al. (1998) and Jack (2001) explain that insurers can reduce the health expenditure levels thanks to some specific actions. For example, insurers can influence their policy holders' consumption behavior by imposing the system of a gatekeeper which prevents them from consulting directly the specialists, by negotiating tariffs with suppliers, by monitoring the prescriptions of doctors,... So, in our model, we shall consider that the cost functions of insurers are influenced by an effort variable.

Actually, the fact that insurers' cost functions depend on the effort they make to reduce the level of health expenditure implies that we must restrict our attention to auctions of incentive contracts. So a regulation through procurement auction in the health insurance sector has two objectives: to select the more efficient insurers and to introduce an incentive scheme to obtain an optimal level of cost reduction effort.

The cost functions of the insurers also depend on the economies of scale which may be present in the health insurance sector. It is the subject of the next paragraph.

Economies of scale and choice of the auction

The assumption of a natural monopoly for health insurance is sometimes mentioned in the literature.[11] The answer to this assumption is crucial to establish a regulation of the health insurance sector built on an auction mechanism. If returns of scale are always increasing, it may seem to be more efficient to create only one group of policy holders and to sell it by auction. But at the same time, big groups may exclude small insurers from the auction procedure so reduce the ex ante competition. Diamond (1992) has already mentioned this problem: "*Groups should be large enough to take advantage of the economies of scale in insurance administration. Groups should be small enough so that most areas will have a number of independent groups, possibly being serviced by different insurance companies.*" Following Diamond's recommendations, the groups of policy holders must be sufficiently big because of economies of scale but sufficiently small to establish a norm of competition and to avoid that

[11] This assumption suggests that management of health risk is less expensive when a single insurer is present. However, this conclusion is often based on the comparison between individual contracts and collective contracts.[12] 038<p type="texpara" tag="Body Text" >For an example : Diamond (1992). In fact, the validity of this assumption is impossible because it implies to compare two organizations of the health insurance which are, by definition, mutually exclusive. So econometric analysis must focus on the measure of returns of scale in the supply of collective insurance.

small companies are automatically excluded. Now one major failure of the optimal auction theory is that in a multi-unit case the size of the shares (here groups) is not determined by the auction mechanism so it implies to determine them arbitrarily.

This problem can be solved thanks to split award auctions. The regulator constitutes several groups, not too big, so as to ensure enough competition by avoiding that small insurers are automatically excluded. After that, the regulator allows the insurers to supply all possible combinations. So, the insurers can make bids to insure one or several groups of policy holders constituted by the regulator. In this case, it is obvious that economies of scale will play a crucial role. If an insurer benefits from scale economies, the sum of the profitability obtained by two different groups will not be equal to the profitability of a contract for the management of the two groups. Intuitively, if the health insurance sector exhibits increasing returns of scale, then only one insurer must manage all the groups. On contrary, decreasing returns of scale tends to favor splitting. Nevertheless, because of the asymmetric information about true cost, the optimal choice cannot be done *a priori*. Contrary to traditional auction mechanisms (mono-object, uniform, discriminatory ones), split award auctions permit to take into account the interaction between all the combinations due to economies of scale (McGuire and Riordan (1995), Dana and Spier (1994)).

To sum up, we have explained that four points must be taken into account if public authorities wish to regulate health insurance markets through auction mechanisms. First, the cost functions of insurers depend on the aggregated levels of the groups of policy holders, which can be supposed common knowledge. It also depends on a privately known efficiency parameter and an unobservable effort variable, which imposes to consider auctions of incentive contracts. Finally, the total health expenditure depends on returns of scale. In order to avoid the exclusion of small insurers from the competition, the regulator must constitute several groups of policy holders which imply to consider split award auctions. We shall develop in the following section a theoretical model of split award auction of incentive contracts.

THE AUCTION DESIGN

The model that we propound is based on the analysis by Laffont and Tirole (1991) which deals with the auctioning of incentive contracts and split award auctions of independent contracts. In our analysis, we specifically define the health insurance cost structure.

We begin this section by giving the main assumptions. We give the allocations under perfect information. After that, we outline and solve the mechanism design problem faced by the regulator when insurers have private information about their productivity level and when their efforts of cost reduction are unobservable. To finish, we characterize a concrete implementation of the optimal mechanism.

Assumptions

In order to simplify the analysis, we consider that the regulator only creates two homogenous groups of policy holders and decides to sell them by auction. This assumption permits to simplify the analysis without affecting the generality of our proposition. In this context, the groups of policy holders are sold to the same insurer or to two different ones. There are n insurers i, $i = 1, ..., n$, which are able to insure the two groups of policy holders. So, there are two possible allocations: split (s) or grouped (m):

$$\chi = \{s, m\}$$

These different allocations imply different cost structures:

$$\begin{aligned} C_m^i &= 2\alpha\theta(\beta_i - e_m^i) \ \forall i = 1, ...n \\ C_s^i &= \theta(\beta_i - e_s^i) \end{aligned}$$

Besides, we shall use the following notation:

$$\begin{aligned}\kappa_m &= 2\alpha\theta \\ \kappa_s &= \theta\end{aligned}$$

The managers of insurance companies can make some efforts e^i to reduce the health expenditure level of their policy holders. These efforts cause the managers a desutility represented by the function $\phi(e^i_k)$ with $\phi' > 0$, $\phi'' > 0$, $\phi''' \geq 0$, and $\phi(0) = 0$.[13]

The efficiency parameters β^i are drawn independently from the same cumulative distribution function $F(.)$ on the interval $B = [\underline{\beta}, \overline{\beta}]$ and a differentiable density function $f(.)$ that is bounded by a strictly positive number on $[\underline{\beta}, \overline{\beta}]$. These two functions are assumed being common knowledge and satisfy the traditional monotone hazard rate property.

The regulator can observe ex post of the health expenditure level and we assume that he/she knows the return of scale (α) effective in the health insurance sector.

We note $x^m_i(\beta)$ the probability that an insurer i is chosen to manage the health risks of the two groups of policy holders and $x^s_i(\beta)$ the probability that i manages only one of the two groups. The feasibility constraint implies:

$$\sum_{i=1}^{n} (x^m_i + x^s_i) \leq 1 \tag{1}$$

Regulation with full information

We begin to study the case in which the regulator knows the levels of the efficiency parameters and can observe the amounts of effort realized. Auctions of incentive contracts implies that the policy holders' health expenditure are reimbursed to the insurers and that they receive an additional transfert t^i for the effort desutility. The utility of the insurers is also the difference between this additional transfert and

[13] c.f. Laffont et Tirole (1987).

the effort desutility. According to the assumption of standardization of the service provided by insurers, the policy holders' surplus is a simple constant S. Contributions are collected by the regulator and distributed to the insurers through the transfer rules specified by auction mechanism. So we consider a parameter λ which captures the social cost of public funds. If the regulator is characterized by an utilitarian objective, his program is:

$$\begin{cases} \max\limits_{x,U,e}(\sum_{i=1}^{n} x_i^m + \frac{1}{2}\sum_{i=1}^{n} x_i^s)S - \lambda\sum_{i=1}^{n} U^i - (1+\lambda)[\sum_{i=1}^{n}\sum_{k\in\chi} x_i^k(\kappa_k(\beta_i - e_k^i) + \phi_k^i(e_k^i))] \\ s.c.\ U^i = t^i - \phi^i(e^i)) \geq 0 \\ \sum_{i=1}^{n}(x_i^m + x_i^s) \leq 1 \\ x_i^k \geq 0\ \forall i = 1, ...n, \forall k \in \chi \end{cases}$$

The solutions to this program are:

$$\begin{aligned} \phi_k^{i'}(e_k^i) &= \kappa_k \text{ so } e_k^i = e_k^* \\ U^i &= 0 \end{aligned}$$

In this context, the regulator selects the two insurers which are characterized by the best efficiency parameters (β_1, β_2) and determines the allocations of the groups according to the sense of the following inequalities:

$$2\alpha\theta(\beta_1 - e_m^*) + \phi(e_m^*) \gtreqless \theta(\beta_1 - e_s^*) + \phi(e_s^*) + \theta(\beta_2 - e_s^*) + \phi(e_s^*) \quad (2)$$

If we focus our attention on a quadratic function of desutility,

$$\phi(e) = \frac{e^2}{2d}$$

we obtain:

$$\begin{aligned} e_m^* &= d2\alpha\theta \\ e_s^* &= d\theta \end{aligned}$$

It should be noted that the optimal effort level differs whether the structure is monopoly or split. In a quadratic case, the last inequality becomes:

$$2\alpha\beta_1 \gtrless (\beta_1 + \beta_2) - d\theta(1 - 2\alpha^2) \tag{3}$$

Three different effects can be made out this selection rule.

- The first effect, that we named *productivity effect*, can be appreciated more easily if we strike the effort variable off the selection rule. In this case, the selection rule would only depend on the comparison between $2\alpha\beta_1$ and $(\beta_1+\beta_2)$. We can observe that the probability to select only one insurer to manage the two groups of policy holders is an increasing function of the return of scale (which is inversely proportional to α). Indeed, the monopoly structure is chosen if $(2\alpha - 1)\beta_1 < \beta_2$. The connection between the return of scale and the choice of the structure is more intuitive when we consider the most favorable case for splitting, $\beta_2 = \beta_{1.}$ In this case, the monopoly structure will always be chosen if $\alpha < 1$. If the health insurance sector is characterized by increasing returns of scale, in a complete information case where cost reduction efforts are ignored, the regulator's situation is improved by awarding the whole market to a single insurer.

- The second constituent is a *cost-reduction effect*. If insurers can make some efforts to reduce the health expenditure levels of their policy holders, then the selection rule must be distorted so as to take account of the impact of these efforts. If we isolate this effect from the selection rule, the monopoly structure is chosen if:

$$
\begin{aligned}
4\alpha^2\theta^2 d &> 2d\theta^2 \\
&\Leftrightarrow \\
\alpha &> \frac{\sqrt{2}}{2}
\end{aligned}
\tag{4}
$$

It should be noted that this effect is counterintuitive because according to optimal effort level difference, monopoly structure is favorised as long as return of scale are not too high. Even in a complete information setting, introducing effort on health expenditure reduction tends to moderate the first *productivity effect*.

- At this *cost reduction effec*t, we must add the second consequence of the effort variable: the *effort desutility effect*. In our quadratic case, we can observe that increasing returns of scale favour the selection of a monoply structure.

$$
\begin{aligned}
2d\alpha^2\theta^2 &< d\theta^2 \\
&\Leftrightarrow \\
\alpha &< \frac{\sqrt{2}}{2}
\end{aligned}
\tag{5}
$$

To finish with this complete information structure, selection rule must take into account the tradeoff between three effects. The *productivity effect* is very intuitive and implies that monopoly structure is favorised by increasing return of scale. The influence of the effort variable on the selection rule is more complex and respectively depends on the weight of the two last effects that we have just described.

Now that we analyze the selection rule in the perfect information structure, we must consider the more realistic case of incomplete information.

Regulation with incomplete information

In this information structure, the efficiency parameters are private information of the insurers and the efforts that they undertake are unobservable by the regulator. Then, the efficiency of an insurer enables him/her to evaluate the profitability generated by the management of groups of policy holders. The regulator must also propound a selection rule and an incentive scheme in order to reduce inefficiencies caused by this information structure. Using the revelation principle, we can view this regulatory procedure as a revelation mechanism $\left\{ \left(x_i^m(\beta)\right), \left(x_i^s(\beta)\right) \left(C_k^i(\beta)\right)_{k\in\chi}, \left(t^i(\beta)\right), \forall i=1,...n \right\}$ where $C_k^i(\beta)$ is the level of health expenditure that the regulator demands to the insurers if they announce β.

Insurer's bidding behavior.—

Ex ante, insurer i's expected utility is:

$$\underset{\beta^{-i}}{E}\left[t^i(\hat{\beta}) - x_i^m(\hat{\beta})\phi(e_m^i) - x_i^s(\hat{\beta})\phi(e_s^i)\right] \ \forall i=1,...n \tag{6}$$

Thanks to the observability ex post of the level of health expenditure, we can write:

$$U^i(\hat{\beta}) = \underset{\beta^{-i}}{E}\left[t^i(\hat{\beta}) - x_i^m(\hat{\beta})\phi(\hat{\beta}^i - \frac{C_m^i(\hat{\beta})}{2\alpha\theta}) - x_i^s(\hat{\beta})\phi(\hat{\beta}^i - \frac{C_s^i(\hat{\beta})}{\theta})\right] \ \forall i=1,...n \tag{7}$$

If we restrict our attention to differentiable mechanisms, the envelope theorem enables us to write necessary conditions for the revelation of the true efficiency parameters. So,

$$\overset{\bullet}{U}{}^i(\beta^i) = -\underset{\beta^{-i}}{E}\left[x_i^m(\beta)\phi'(\hat{\beta}^i - \frac{C_m^i(\hat{\beta})}{2\alpha\theta}) - x_i^s(\beta)\phi'(\hat{\beta}^i - \frac{C_s^i(\hat{\beta})}{\theta})\right] \ \forall i=1,...n \tag{8}$$

Optimal auction.—

The maximand of an utilitarian regulator is:

$$(\sum_{i=1}^n x_i^m + \frac{1}{2}\sum_{i=1}^n x_i^s)S - (1+\lambda)\sum_{i=1}^n t^i - (1+\lambda)[\sum_{i=1}^n \left(x_i^m C_i^m + x_{ij}^s C_i^s\right)]$$

which can be rewritten:

$$(\sum_{i=1}^{n} x_i^m + \frac{1}{2}\sum_{i=1}^{n} x_i^s)S - \lambda \sum_{i=1}^{n} U^i - (1+\lambda)[\sum_{i=1}^{n} \left(x_i^m(C_i^m + \phi_m^i) + x_i^s(C_i^s + \phi_s^i)\right)] \quad (9)$$

Under incomplete information, the regulator maximizes collective welfare subject to incentives, individual rationality and feasibility constraints. So, the regulator's optimization problem is:

$$\begin{cases} \max\limits_{(x,C,U)} \underset{\beta}{E}(\sum_{i=1}^{n} x_i^m(\beta) + \frac{1}{2}\sum_{i=1}^{n} x_i^s(\beta))S - \lambda \sum_{i=1}^{n} U^i(\beta^i) \\ -(1+\lambda)[\sum_{i=1}^{n}\sum_{k\in\chi} x_i^k(\beta)(C_k^i(\beta) + \phi^i(\beta^i - \frac{C_k^i(\beta)}{\kappa_k})] \\ s.t. \\ \overset{\bullet}{U}{}^i(\beta^i) = -\underset{\beta^{-i}}{E}\left[\sum_{k\in\chi} x_i^k(\beta)\phi'(\hat{\beta}^i - \frac{C_k^i(\hat{\beta})}{\kappa_k})\right] \ \forall i = 1,...n \\ U^i(\overline{\beta}^i) = 0 \ \forall i = j,k,l \\ \sum_{i=1}^{n}(x_i^m + x_i^m) \leq 1 \\ x_i^k \geq 0 \ \forall i = 1,...n, \forall k \in \chi \end{cases}$$

As follows, we shall note:

$$X_i^k(\beta^i) = E_{\beta^{-i}}[x_i^k(\beta)] \ \forall i = 1,...n, \forall k \in \chi \quad (10)$$

Laffont and Tirole (1987) explain that this problem can be solved in two steps. Maximizing pointwise with respect to $C(.)$ implies that the optimal effort levels required from the insurers are solutions of:

$$\frac{\lambda}{1+\lambda}\frac{F_i(\beta^i)}{f_i(\beta^i)}\phi''(\beta^i - \frac{C_k^{*i}(\beta^i)}{\kappa_k}) = [\kappa_k - \phi'(\beta^i - \frac{C_k^{*i}(\beta^i)}{\kappa_k})] \quad (11)$$

We can easily observe that optimal effort levels are decreasing functions of the efficiency parameters. From the above equation, we obtain:

$$e_m^*(\beta^i) > e_s^*(\beta^i)$$

In the case of a quadratic function, we have:

$$\begin{aligned} \forall i &= 1,...n,\ \forall k \in \chi \\ e_k^*(\beta^i) &= d\kappa_k - \frac{\lambda}{1+\lambda}\frac{F_i(\beta^i)}{f_i(\beta^i)} \end{aligned} \tag{12}$$

This optimal effort function reflects the hypothesis of unobservable effort of reducing the individual cost. Informational rents $(\frac{\lambda}{1+\lambda}\frac{F_i(\beta^i)}{f_i(\beta^i)})$ are so independent from the structure (κ_k). It is now interesting to compare the levels of efforts according to the modality of allocation (s or m). Thus,

$$\begin{aligned} 2e_s^*(\beta^i) &\leq e_m^*(\beta^i) \\ &\Leftrightarrow \\ \frac{\lambda}{1+\lambda}\frac{F_i(\beta^i)}{2d\theta f_i(\beta^i)} &\geq 1-\alpha \end{aligned}$$

If we have some decreasing returns of scale in the health insurance industry ($\alpha \geq 1$), then the effort that an insurer must undertake is more important when he/she is in a monopoly situation (it should be noted that the left member of the inequality is always positive and $1-\alpha \leq 0$ for $\alpha \geq 1$).

Remember that the incentive constraint for each insurer is:

$$\overset{\bullet}{U}{}^i(\beta^i) = -\underset{\beta^{-i}}{E}\left[x_i^m(\beta)\phi'(e_m^*(\beta^i)) + x_i^s(\beta)\phi'(e_s^*(\beta^i))\right]$$

As McGuire and Riordan (1995), we notice that this equation reflects the two opposite effects of splitting the health insurance sector. First of all, it reduces the insurer rent as $\phi'(e_m^*(\beta_1)) > \phi'(e_s^*(\beta_1))$, but raises the second insurer selected as $\phi'(e_s^*(\beta_2)) > 0$. This is the *cushion effect* which decreases the cost of not being selected alone (Laffont and Tirole, 1991).

If we consider the optimal value $C_k^{*i}(\beta^i)$, $\forall i = 1,...n$, $\forall k \in \chi$ as given, using traditional development, we can determine the optimal selection rule. The objective function can be rewritten,

$$\int_B [\sum_{i=1}^n x_i^m(\beta) + \frac{1}{2}\sum_{i=1}^n x_i^s(\beta))S - \sum_{i=1}^n \sum_{k\in\chi} x_i^k(\beta)(1+\lambda)[\kappa_k(\beta^i - e_k^{i*}(\beta^i)) + \phi(e_k^{i*}(\beta^i))$$
$$+\frac{\lambda}{1+\lambda}\frac{F_i(\beta^i)}{f_i(\beta^i)}\phi'(e_k^{i*}(\beta^i)))]dF(\beta)$$

To be selected on a specific allocation, an insurer must candidate for this allocation and announce the best efficiency parameter.

We consider the following traditional virtual values:

$$r_k^i(\beta^i) = \kappa_k(\beta^i - e_k^{i*}(\beta^i)) + \phi(e_k^{i*}(\beta^i)) + \frac{\lambda}{1+\lambda}\frac{F_i(\beta^i)}{f_i(\beta^i)}\phi'(e_k^{i*}(\beta^i)) \tag{13}$$

An insurer i will be selected for the two groups of policy holders if:

$$r_m^i(\beta^i) < r_s^i(\beta^i) + r_s^j(\beta^j)\ \forall j \neq i$$

The rule determining the optimal structure deserves some comments. It should be noted that, for a small social cost of the public funds , the efforts are near first best (complete information) levels and the government must concede an additional informational rent when the two insurers are selected. This tends to favor, in incomplete information settings, the monopoly structure. Nevertheless, for large values of λ, effort levels are far lower than in complete information, and this may work in favor of splitting the health insurance sector. The following, with a quadratic desutility function, illustrates the impact of asymmetric information.

Notice $v(\beta^i) = \beta^i + \frac{\lambda}{1+\lambda}\frac{F(\beta^i)}{f(\beta^i)}$. The appropriate structure is chosen according to the result of the last inequality:

$$\begin{aligned}
& 2\alpha\theta(\beta^i + \frac{\lambda}{1+\lambda}\frac{F(\beta^i)}{f(\beta^i)}) - 2d\alpha^2\theta^2 - \frac{\left(\frac{\lambda}{1+\lambda}\frac{F(\beta^i)}{f(\beta^i)}\right)^2}{2d} \\
< \quad & \theta(\beta^i + \frac{\lambda}{1+\lambda}\frac{F(\beta^i)}{f(\beta^i)}) - \frac{d\theta^2}{2} - \frac{\left(\frac{\lambda}{1+\lambda}\frac{F(\beta^i)}{f(\beta^i)}\right)^2}{2d} \\
& +\theta(\beta^j + \frac{\lambda}{1+\lambda}\frac{F(\beta^j)}{f(\beta^j)}) - \frac{d\theta^2}{2} - \frac{\left(\frac{\lambda}{1+\lambda}\frac{F(\beta^j)}{f(\beta^j)}\right)^2}{2d}
\end{aligned}$$

Consider the most favorable case for splitting, i.e. $\beta^i = \beta^j$. Then, the informational cost of selecting a sole insurer is given by

$$\frac{\lambda}{1+\lambda}\frac{F}{f}(2\alpha\theta - \frac{\left(\frac{\lambda}{1+\lambda}\frac{F}{f}\right)}{2d})$$

instead of

$$\frac{\lambda}{1+\lambda}\frac{F}{f}(2\theta - \frac{\left(\frac{\lambda}{1+\lambda}\frac{F}{f}\right)}{d})$$

in splitting structure. Depending on the value of α, this may favor the monopoly structure, if

$$\alpha < 1 - \left(\frac{\lambda}{1+\lambda}\frac{F}{4d\theta f}\right)$$

The last inequality implies that a monopoly structure is chosen if returns of scale are **sufficiently increasing**. Indeed, splitting may be prefered even though $\alpha < 1$. We can also conclude that, all things equall, the incomplete information structure tends to favour splitting. Besides, as in the perfect information case, it is interesting to reveal the three effects who compose the selection rule and to explain the consequences of the asymetric information for each effect.

Firstly, consider the *productivity effect*. We can observe that the sense of this effect is similar to the perfect information case:[14]

$$(2\alpha - 1)v(\beta_1) < v(\beta_2) \tag{14}$$

The difference between the two information structures is that, in the incomplete case, the productivity effect is ponderated by the informational rents through the traditional rate hazard:

$$(2\alpha - 1)\frac{F(\beta_1)}{f(\beta_1)} < \frac{F(\beta_2)}{f(\beta_2)} \tag{15}$$

[14]For comodity, we shall use the inequality in the sense of the monopoly structure selection.

The sense of the inequality for *the cost reduction* effect is the same as in the perfect information case:

$$4\alpha^2\theta^2 d - 2\alpha\theta\frac{\lambda}{1+\lambda}\frac{F(\beta_1)}{f(\beta_1)} > 2d\theta^2 - \theta\frac{\lambda}{1+\lambda}\left(\frac{F(\beta_1)}{f(\beta_1)} + \frac{F(\beta_2)}{f(\beta_2)}\right)$$

even though, the *cost reduction effect* is reduced according to the informational rents, given by (15).

The impact of the *effort desutility effect* can be isolate too. It contributes to favour monopoly structure if:

$$\frac{\left(d2\alpha\theta - \frac{\lambda}{1+\lambda}\frac{F(\beta_1)}{f(\beta_1)}\right)^2}{2d} < \frac{\left(d\theta - \frac{\lambda}{1+\lambda}\frac{F(\beta_1)}{f(\beta_1)}\right)^2}{2d} + \frac{\left(d\theta - \frac{\lambda}{1+\lambda}\frac{F(\beta_2)}{f(\beta_2)}\right)^2}{2d} \tag{16}$$

As for the two preceedings effects, the difference with the perfect information case depends on the informational rents less a residue:

$$(2\alpha - 1)\frac{F(\beta_1)}{f(\beta_1)} > \frac{F(\beta_2)}{f(\beta_2)} - \frac{\frac{\lambda}{1+\lambda}\left(\frac{F(\beta_2)}{f(\beta_2)}\right)^2}{2d\theta} \tag{17}$$

To conclude, we can remark that incomplete information structure does not change the sense of the selection rule with regard to returns of scale. The three effects that we reveal in the perfect information structure still exist in the incomplete information structure. They are only emphasized (*productivity effect*) or diminished (the two efforts effects) by the informational rents. Incomplete informational structure is not neutral anyway because returns of scale must be higher than in the perfect case to select only one insurer.

Implementation by a dominant strategy auction.—

Implementation by a dominant strategy auction crucially depends on the monotonicity of the selection rules. Following Dana and Spier (1994), we prove that such conditions are satisfied in the case of quadratic desutility of effort function (see appendix). For the following, we suppose monotonicity satisfied.

Let t^{i*} be the insurer i's expected transfer as a function of the announced bids. We have,

$$\begin{aligned}
t^{i*} &= \underset{\beta^{-i}}{E}\, t^{i*}(\beta) = U^{*i}(\beta^i) + X_i^{m*}(\beta^i)\phi\left(\beta^i - \frac{C_m^{i*}(\beta^i)}{2\alpha\theta}\right) + X_i^{s*}(\beta^i)\phi(\beta^i - \frac{C_s^{i*}(\beta^i)}{\theta}) \\
&= \int_{\beta^i}^{\overline{\beta}} X_i^{m*}(\widetilde{\beta}^i)\phi'\left(\widetilde{\beta}^i - \frac{C_m^{i*}(\widetilde{\beta}^i)}{2\alpha\theta}\right) d\widetilde{\beta}^i + \int_{\beta^i}^{\overline{\beta}} X_i^{s*}(\widetilde{\beta}^i)\phi'\left(\widetilde{\beta}^i - \frac{C_s^{i*}(\widetilde{\beta}^i)}{\theta}\right) d\widetilde{\beta}^i \\
&\quad + X_i^{m*}(\beta^i)\phi\left(\beta^i - \frac{C_m^{i*}(\beta^i)}{2\alpha\theta}\right) + X_i^{s*}(\beta^i)\phi(\beta^i - \frac{C_s^{i*}(\beta^i)}{\theta}) \qquad (18)
\end{aligned}$$

Then, the optimal selection rule is:

$$\begin{aligned}
x_m^{i*}(\beta^i) &= 1 \Leftrightarrow \begin{cases} r_m(\beta^i) < r_s(\beta^i) + r_s(\max\limits_{j\neq i}\beta^j) \\ \beta^i < \max\limits_{j\neq i}\beta^j \end{cases} \\
&\Leftrightarrow \begin{cases} \beta^i < r_{m-s}^{-1}(r_s(\max\limits_{j\neq i}\beta^j)) \\ \beta^i < \max\limits_{j\neq i}\beta^j \end{cases} \\
&\Leftrightarrow \beta^i < r_{m-s}^{-1}(r_s(\max\limits_{j\neq i}\beta^j))
\end{aligned}$$

Thanks to the selection rule, the threshold values can be determined. Let β^2 and β^3 be respectively the first and the second lowest efficiency parameters among the $n-1$ insurers different from i. The transfers can be rewritten:

$$t_m^{i*} = \phi\left(\beta^i - \frac{C_m^{i*}(\beta^i)}{2\alpha\theta}\right) + \int_{\beta^i}^{r_{m-s}^{-1}(r_s(\beta^2))} \phi'\left(\widetilde{\beta}^i - \frac{C_m^{i*}(\widetilde{\beta}^i)}{2\alpha\theta}\right) d\widetilde{\beta}^i \qquad (19)$$

And, if $\beta^i < \beta^2$

$$t_s^{i*} = \phi\left(\beta^i - \frac{C_s^{i*}(\beta^i)}{\theta}\right) + \int_{\beta^i}^{r_{m-s}^{-1}(r_s(\beta^2))} \phi'\left(\widetilde{\beta}^i - \frac{C_m^{i*}(\widetilde{\beta}^i)}{2\alpha\theta}\right) d\widetilde{\beta}^i \qquad (20)$$

if $\beta^2 < \beta^i < \beta^3$

$$t_s^{i*} = \phi\left(\beta^i - \frac{C_s^{i*}(\beta^i)}{\theta}\right) + \int_{\beta^i}^{r_s^{-1}(\min(r_{m-s}(\beta^1),\beta^3))} \phi'\left(\widetilde{\beta}^i - \frac{C_m^{i*}(\widetilde{\beta}^i)}{2\alpha\theta}\right) d\widetilde{\beta}^i$$

The following proposition gives the implementation of the optimal mechanism in dominant strategies.

Proposition 1 *The optimal mechanism can be implemented using the following procedure:*

i. Insurers transmit bids $b^i = \beta^i$*.*

ii. The mechanism preselects insurers 1 and 2, with $b^1 = \min(b^i)$ *and* $b^2 = \min(b^i, i \neq 1)$*.*

ii. The bids are compared with the functions $r_{m-s}^{-1}(r_s(.))$ *and* $r_s^{-1}(r_s(.) - r_m(.))$*.*

iii. If $b^1 < r_{m-s}^{-1}(r_s(b^2))$*, insurer 1 wins the whole market. The regulator determines a global cost objective* $C_m^{i*}(\beta^1)$ *and transfers him/her.*

$$\phi\left(\beta^i - \frac{C_m^{i*}(\beta^i)}{2\alpha\theta}\right) + \int_{\beta^i}^{r_{m-s}^{-1}(r_s(\beta^2))} \phi'\left(\beta^i - \frac{C_m^{i*}(\beta^i)}{2\alpha\theta}\right) d\widetilde{\beta}^i$$

If insurer 1 does not respect the cost objective, he/she must support part of the additional cost given by:

$$d_m(b^1)[C - C_m^*(b^1)] \text{ with } d_m(b^1) = \phi'\left(\beta^i - \frac{C_m^{i*}(\beta^i)}{2\alpha\theta}\right).$$

iv. If $b^1 > r_{m-s}^{-1}(r_s(b^2))$*, the health insurance sector is split between insurer 1 and insurer 2, who have to respect a cost objective of* $C_s^{i*}(\beta^1)$ *and* $C_s^{i*}(\beta^2)$*. The regulator transfers respectively*

$$\phi\left(\beta^1 - \frac{C_s^{i*}(\beta^1)}{\theta}\right) + \int_{\beta^i}^{r_{m-s}^{-1}(r_s(\beta^2)} \phi'\left(\beta^i - \frac{C_s^{i*}(\beta^1)}{\theta}\right) d\widetilde{\beta}^i$$

$$\phi\left(\beta^2 - \frac{C_s^{i*}(\beta^2)}{\theta}\right) + \int_{\beta^i}^{r_s^{-1}(\min(r_{m-s}(\beta^1),\beta^3))} \phi'\left(\beta^i - \frac{C_s^{i*}(\beta^2)}{\theta}\right) d\widetilde{\beta}^i$$

and defines a sharing of additional cost respectively:

$$d_s(b^1)[C - C_s^*(b^1)] \text{ with } d_s(b^1) = \phi'\left(\beta^2 - \frac{C_s^{i*}(\beta^2)}{\theta}\right)$$

$$d_s(b^2)[C - C_s^*(b^2)] \text{ with } d_s(b^2) = \phi'\left(\beta^2 - \frac{C_s^{i*}(\beta^2)}{\theta}\right)$$

(We demonstrate in the appendix that the truth telling is a dominant strategy in the auction.)

This implementation deserves some comments. Contrary to Dana and Spier (1994), our auctioning proposal does not imply ex post competition for collective contracts, because the selected insurers manage their policy holders' groups as a monopoly. Nevertheless, the mechanism introduces ex ante competition during the selection process. Furthermore, our endogeneous selected structure generates specific competition effects. Each insurer competes directly with each other but the comparison of the different structures (monopoly or split) generates internal indirect competition. Besides, this mechanism is resistant to cost disturbance (Laffont and Tirole (1987)), which seems to be well adapted to the health insurance, where health expenditure is partly random. Finally it should be pointed out that the optimal mechanism implies a differentiated treatment of insurers when more than one of them are selected. Cost objectives, initial transfers and sharing of the additional cost depend on the revealed efficiency of the insurers.

CONCLUSION

In this paper, we completed Diamond's suggestion (1992) which propounds to introduce auctions to regulate competition in the health insurance sector. This type of regulation has the advantage of orienting the competitive pressure and permits to avoid the traditional inefficiencies of the individual health insurance markets. We did not expose a precise institutional environment where auction could be implemented. We defined the main characteristics of the organization of the health insurance sector that the utilization of the auction mechanism would require. These characteristics are the modalities of formation of policy holder groups, the funding of the health expenditure, the definition of a standard quality service to avoid the distortion of ex ante competition, which also implies the definition of a pannier of care.

We set out in the first section the kind of auction that the health insurance sector's characteristics would require. We explained that the auction must consider the fact that insurers can carry out some actions to reduce the level of health expenditure of their policy holders and that scale economies in the health insurance sector must influence the selection rule in the auction. Our description of these specificities permitted us to concentrate the analysis on split award auctions of incentive contracts.

Even though the literature of the auction theory is very abundant, few articles deal with split-award auction of incentive contracts. Then, we propounded a formal model to determine the optimal auction according to our specific assumptions about the cost structure. We focused our attention on the specific case where only two groups of policy holders are considered, the generalization to n groups introducing technicalities without modifying the main results of the analysis. Our findings showed that the results of the auction crucially depend on the scale economies of the health insurance sector, but that adverse selection and moral hazard exhibit counter-intuitive effects.

Our analysis can be applied to different situations. If we extend Diamond's suggestion, the split award auction of incentive contracts can be introduced to regulate or organize more efficiently competition in the collective health insurance markets. We explained in the introduction that a non-regulated health insurance market has some failures that ex ante competition can avoid. To resume Diamond's idea, if auction mechanisms are used in the United States to regulate the health insurance industry, the regulator must also take into account the interaction between the auctions in different states.

Auctions can also be used to introduce competition in a public health insurance system. The public insurer who was in a monopoly situation can take part in ex ante competition as private insurers can. In this case, the auction procedure can take into account the fact that the public insurer can make before the auction some investments to improve his productivity in the health insurance. The optimal mechanism then depends on the transferability of the investments.[15]

REFERENCES

[1] Blair R.D, Ginsburg P.B, and Vogel R.J, 1975, "Blue Cross-Blue Shield Administrative Costs: A study of Non-Profit Insurers", *Economic Inquiry*,p237-251.

[2] Couffinhal A., Henriet D. and Rochet J.C, 2000, "The impact of Public Health Insurance Systems on Access to Health Care and Labor Force Participation: A Theoretical Analysis", Working Paper Credes-Greqam-Gremaq.

[3] Cutler D. and Reber S., 1998, "Paying for Health Insurance: The trade-off between Competition and Adverse Selection", *Quaterly Journal of Economics*, 113(2), p433-466.

[4] Dana J.D. and Spier K.E, 1994, "Designing a Private Industry: Government auctions with endogenous market structure", *Journal of Public Economics*, 53, p127-147.

[5] Diamond P., 1992, "Organizing the Health insurance Markets", *Econometrica*, vol 60(6), p1233-1254.

[6] Henriet D. and Rochet J.C, 1999, "The Political Economy of the Health Insurance", Working Paper Greqam-Gremaq.

[7] Jack W., 1999, "Optimal Redistributive Taxation when Individual differ on two dimensions: The Integration of Health Expenditure into a Linear Income Tax, paper presented at the International Colloque d'Economie Publique, Université de Bretagne Occidentale, France.

[15] Cf Laffont and Tirole (1988).

[8] Jack W., 2001, "Controling Selection Incentives when Health Insurance Contracts are endogenous", *Journal of Public Economics*, 80, p25-48.

[9] Klemperer P., 1999, "Auction Theory: A Guide to the Literature", *Journal of Economics Survey*, vol 13(3), p227-286.

[10] Laffont J.J and Tirole J., 1987, "Auctionning of Incentives Contracts", Journal of Political Economy, 95(5), p921-937.

[11] Laffont J.J and Tirole J., 1988, "Repeated Auctions of Incentives Contracts, Investment and Bidding Parity, with an Application to Takeovers", *Rand Journal of Economics*, 19, p516-537.

[12] Laffont J.J and Tirole J., 1991, *A Theory of Incentives in Procurement and Regulation*, MIT Press, Cambridge, MA.

[13] McGuire T. and Riordan M., 1995, "Incomplete Information and Optimal Market Structure: Public Purchases from Private Providers", *Journal of Public Economics*, 56, p125-141.

[14] Pauly M.V, 1984, "Is Cream Skimming a Problem for the Competitive Medical Market ?", *Journal of Health Economics*, 3, p87-95.

[15] Pauly M.V, Danzon P., Feldstein P. and Hoff J., 1991, "A Plan for Responsible National Health Insurance", *Health Affairs*, 10(1), p5-25.

[16] Pauly M.V, 2000, "Optimal Health Insurance", *The Geneva Papers of Risk and Insurance*, vol 25(1), p116-127.

[17] Riordan M. and Sappington D., 1987, "Awarding Monopoly Franchises", *American Economic Review*, 77, p375-387.

[18] Rochet J.C, 1991, "Incentives, Redistribution and Social Insurance ", *The Geneva Papers of Risk and Insurance Theory*, 16, p143-165.

[19] Rothschild M. and Stiglitz J., 1976, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information", *Quaterly Journal of Economics*, 90, p629-649.

[20] Schokkaert E., Dhaene G. and Van de Voorde C., 1998, "Risk Adjustment and the Trade-off between Efficiency and Risk Selection ; an Application of the Theory of fair Compensation", *Health Economics,* 7, p465-480.

APPENDIX

Proof of the implementation by dominant strategies

We verify in this appendix that truth-telling is a dominant strategy in our modified second-price auction. We must consider several cases: when the true efficiency parameter of the insurer i belongs to the interval of the monopoly, when it belongs to the "*split interval*" and finally when it belongs to the interval where insurers should be not selected.

Before that, as Laffont and Tirole (1987) remarked, we can see that the individual rationality of the insurers implies that they will not lie if their falsehood prevents them from being selected (because in this case, they will win nothing). We note β^i the true parameter and b^i the announcement of the insurer i. So, consider that an insurer i makes an announcement b^i such that $b^i < \min_k b^k$. His objective is:

$$\max_{b^i < \min b^k} \left\{ t^i(b^i, b^{-i}) - \phi\left(b^i - C^{i*}_{\kappa_k}(b^i)\right)\right\} \tag{21}$$

The first order condition is:

$$\phi'\left(b^i - C^{i*}_{\kappa_k}(b^i)\right)\left(1 - \frac{dC^{i*}}{\beta^i}\right) - \phi\left(b^i - C^{i*}_{\kappa_k}(b^i)\right) + \phi'\left(\beta^i - C^{i*}_{\kappa_k}(b^i)\right)\frac{dC^{i*}}{\beta^i} = 0$$

It is easy to check that the unique solution is $b^i = \beta^i$, $\forall \kappa_k$. Consider the case where $\beta^i < r_{m-s}^{-1}(r_s(\beta^2))$. We must verify that an insurer has no interest to make an announcement b^i which belongs to the "*split interval*" $(r_{m-s}^{-1}(r_s(\beta^2)) < b^i < \min(r_s^{-1}(r_s(\beta^2) - r_m(\beta^2)), \beta^3))$. If the insurer i lies, his utility becomes:

$$\begin{aligned} U^i(\beta^i, b^i) &= \phi\left(b^i - \frac{C_s^{i*}(b^i)}{\theta}\right) + \int_{b^i}^{\min(r_s^{-1}(r_s(\beta^2)-r_m(\beta^2)),\beta^3)} \phi'\left(\widetilde{b}^i - \frac{C_s^{i*}(\widetilde{b}^i)}{\theta}\right) d\widetilde{b}^i \\ &\quad -\phi\left(\beta^i - \frac{C_s^{i*}(b^i)}{\theta}\right) \end{aligned} \tag{22}$$

It is easy to check that $U^i(\beta^i, b^i) < 0$.[16]

The proof is similar for the other intervals.

Proof of the monotonicity

According to the individual rationality of the insurers, it is straightforward to check that the insurers have no interest to lying when their parameters are inferior to $\min(r_s^{-1}(r_s(\beta^2) - r_m(\beta^2)), \beta^3)$.

Following Dana and Spier (1994), we define the virtual welfare functions:

$$\hat{W}_m(\beta^i) = S - (1+\lambda)\left(2\alpha\theta(\beta^i - e_m^{i*}(\beta^i)) + \phi(e_m^{i*}(\beta^i)) + \frac{\lambda}{1+\lambda}\frac{F(\beta^i)}{f(\beta^i)}\phi'(e_m^{i*}(\beta^i))\right)$$

$$\hat{W}_s(\beta^i, \beta^j) = S - (1+\lambda)\begin{pmatrix} \theta(\beta^i - e_s^{i*}(\beta^i)) + \phi(e_s^{i*}(\beta^i)) + \frac{\lambda}{1+\lambda}\frac{F(\beta^i)}{f(\beta^i)}\phi'(e_s^{i*}(\beta^i)) \\ +\theta(\beta^j - e_s^{j*}(\beta^j)) + \phi(e_s^{j*}(\beta^j)) + \frac{\lambda}{1+\lambda}\frac{F(\beta^j)}{f(\beta^j)}\phi'(e_s^{j*}(\beta^j)) \end{pmatrix}$$

So,

$$\frac{\partial \hat{W}_m(\beta^i)}{\partial \beta^i} = \frac{\begin{array}{c} f(\beta^i)^2(\lambda^2 F(\beta^i) - 2d\alpha\theta(1+\lambda)(1+2\lambda)f(\beta^i)) \\ -\lambda F(\beta^i)(\lambda F(\beta^i) - 2d\alpha\theta(1+\lambda)f(\beta^i))f'(\beta^i) \end{array}}{d(1+\lambda)f(\beta^i)^3}$$

$$\frac{\partial \hat{W}_s(\beta^i, \beta^j)}{\partial \beta^i} = \frac{\begin{array}{c} -d\theta(1+\lambda)(1+2\lambda)f(\beta^i)^3 - \lambda^2 F(\beta^i)^2 f'(\beta^i) \\ +\lambda F(\beta^i)f(\beta^i)(\lambda f(\beta^i) + d\theta(1+\lambda)f'(\beta^i)) \end{array}}{d(1+\lambda)f(\beta^i)^3}$$

[16] See Laffont and Tirole (1991) for a similar proof.

The monotonicity of the selection rule depends on the difference of the partial derivative of the two types of virtual welfare. Then,

$$\begin{aligned}
&\frac{\partial \hat{W}_m(\beta^i)}{\partial \beta^i} - \frac{\partial \hat{W}_s(\beta^i, \beta^j)}{\partial \beta^i} \\
&= (2\alpha - 1)\theta(-1 - 2\lambda + \frac{\lambda F(\beta^i) f'(\beta^i)}{f(\beta^i)^2}) \\
&= (2\alpha - 1)\theta(-1 - \lambda - \lambda(\frac{d}{d\beta^i}(\frac{F_i(\beta^i)}{f_i(\beta^i)}))
\end{aligned}$$

It is easy to check that the monotonicity of the selection rule is verified if $\alpha > 0.5$.