Axioms for health care resource allocation

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Abstract

We examine welfare theoretic foundations for solutions to the health care resource allocation problem based on measures of *quality adjusted life-years* (QALYs). Conditions for representing the individuals' preferences by QALY measures, as well as conditions for the existence of a social welfare function being a function of the individuals' QALYs only are established. The QALY measures represent deterministic individual utility; no reference to the expected utility hypothesis and risk neutrality with respect to life-years is made. A solution suggested by Dolan (1998) to the health care resource allocation problem is characterized by an axiom stating that the social welfare of life-years distributions is anonymous with respect to the individuals' health states. By replacing this axiom with an axiom of non-age discrimination, we obtain a characterization of the utilitarian solution to the health care resource allocation problem. If neither of these two axioms are acceptable, under quite mild alternative assumptions there exists a social welfare function which is the sum of power transformations of the individuals' QALYs. The latter result is obtained as an application of a theorem due to Roberts (1980).

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1 Introduction

Priority decisions taken by a health planning authority are of obvious importance to the individuals in the society, and it is desirable that the decisions can be formulated as a consequence of simple and transparent principles of fairness. For a fixed stock of resources, a health planning authority must choose one out of many feasible health care resource allocations each having an impact on the individuals' quality and quantity of life. Likewise, a health planning authority may wish to formulate general principles for the way it allocates extra (lesser) health care to the individuals if the set of feasible health care allocations alters due to changes in budgets, in demographic composition, or in available technology.

The aim of this paper is to formulate an axiomatic characterization of methods for allocation of health care resources directly based on the individuals' preferences for own quality and quantity of life. In this sense our approach complements that of Bleichrodt (1997) who investigates a situation with choice under uncertainty and aggregation of *quality adjusted life-years* (QALYs) in this context. In this paper we do not introduce preferences for health lotteries, and as such we do not make use of any assumptions regarding the individuals' attitudes towards risk. Rather, we are aiming to characterize QALY-based solutions to the health care resource allocation problem without introducing health lotteries and risk neutrality. The deterministic framework for establishing QALY-based individual utilities and QALY-based aggregation methods satisfies two purposes. First, consider the viewpoint formulated by Broome (1993) that *"Uncertainty is a complication rather than an essential part of the problem of valuing lives, and it ought not to be introduced into the analysis earlier than needed."* In its simplest formulation, the health care resource allocation problem contains no explicit element of risk. Here, the individuals' attitudes towards risk cannot be used for calibrating individual and societal utility of health. Thus, from a conceptual point of view, it is useful to obtain a characterization of allocation methods separated from explicitly formulated assumptions on the individuals' attitudes towards risk. Second, the axiomatic characterization of individual preferences aids the formulation of simple axioms of fairness regarding priority setting in the society. We demonstrate that the axioms used to derive QALY-based individual preferences are likely to have very useful parallel formulations for the social welfare ordering (preferences for health care resource allocation within a population). For example, a *life-time continuity* axiom has a direct parallel for the social welfare ordering: *life-time continuity of social preferences*. Likewise, a *life-time scale independence* axiom for individual preferences has a direct parallel with respect to social welfare orderings, the *common life-time scale independence* axiom. Accordingly, one may obtain useful axioms for social welfare orderings by extending principles characterizing individual utilities in a natural way to the social welfare ordering.

In Section 2, the necessary definitions are provided in order to give a general formulation of the health care resource allocation problem under consideration. In Section 3, we derive an axiomatic characterization of QALY-measures as individual utility functions in a deterministic framework. In Section 4, we give sufficient conditions for the existence of a continuous social welfare function being a function of individual QALY-measures only. Directly based on individual preferences, we derive axiomatic characterizations of two specific solutions to the health care resource allocation problem that have been suggested in the literature. Dolan's solution (cf. Dolan, 1998) is characterized by applying a key axiom stating that the social welfare of a distribution of life-years should be independent of (in the sense of anonymity) the individuals' actual health states (Section 4.1). By replacing this particular axiom with an axiom of 'non-age discrimination', we obtain a characterization of the utilitarian solution to the health care resource allocation problem (Section 4.2). If neither of these two axioms are acceptable to the health planning authority (or to a panel of representative individuals), we find under some alternative assumptions, which are quite mild in our particular context, there exists a social welfare function being the sum of power transformations of the individuals' QALYs; Dolan's solution and the utilitarian solution are special cases (Section 4.3). The latter result is obtained as an application of a theorem due to Roberts (1980). Section 5 rounds off with some concluding remarks.

In the remainder of this section, we review recent literature on health care resource allocation[1], and we shall try to formulate our motivation in some more detail.

[1]There is a large literature on this topic and our list of references is not intended to be exhaustive.

1.1 Related literature

To fix ideas, assume that the individuals' preferences for health may be captured by QALY measures

$$q_i(a_i, t_i) = h_i(a_i)t_i,$$

where t_i is life-time and $h_i(a_i)$ is a health state value of the health state a_i. Dolan (1998) assumes that the social welfare ordering for allocation of health states $(a_1, ..., a_n)$ and life-times $(t_1, ..., t_n)$ may be represented by a social welfare function of the form

$$U((a_1, t_1), ..., (a_n, t_n)) = W(q_1(a_1, t_1), ..., q_n(a_n, t_n)),$$

where q_i is the level of QALYs for individual i. Measuring social welfare by unweighted summation of individual QALYs may fail to capture distributive justice. Thus Dolan focuses on a social welfare function of the form

$$W(q_1, .., q_n) = \sum_i \ln q_i.$$

In a comment to Dolan's article[2], Johannesson (1999) argues that *"...the empirical approach suggested by Dolan has no theoretical foundation and should not be used as a basis for aggregating QALYs. The main problem with the approach is that it measures only the altruistic values that individuals attach to other peoples health status and ignores the utility that individuals attach to their own health status."* Johannesson suggests the following alternative framework: Each individual's preferences represented by a utility function $u_i = u_i(q_1, .., q_n)$, a function determined by the level of individual QALYs only, but each individual utility function depends now on the entire distribution of individual QALYs. The modified social welfare function is then defined as the sum of individual utility levels,

$$W(q_1, .., q_n) = \sum_i u_i(q_1, .., q_n).$$

The basic idea is that since the individual utilities already capture the altruistic aspects, the social welfare could be measured by unweighted summation of individual utilities.

[2]See also Dolan's reply (1999).

In Nord (1993), Nord et al. (1999) and Nord (1999) variants of the *cost-value* method are suggested and analyzed; this approach have different characteristics compared to that of Dolan or Johannesson (see Østerdal, 2002). Williams (1997) advocates strongly for a *fair innings* principle for which the use of specific kinds of equity weights is suggested; this imply again another structure of the social welfare function.

In sum, quite a few approaches have been suggested in recent literature. It is, however, very difficult to discuss the superiority of one functional form over the other. For example, different structures of the social welfare function may represent the same underlying social preferences; and it may not be clear for some approaches whether key properties such as the Pareto principle are satisfied. Also, the characteristics of these approaches depend crucially on the underlying assumptions behind the individual QALYs. The objective of this paper is therefore to investigate these issues by taking an axiomatic approach: Instead of comparing functions (with quite complicated properties) we may instead compare the underlying axioms which in certain combinations lead to different allocation methods. Besides some mild technical assumptions (for example that preferences are continuous in life-time) we find that key allocation methods boils down to being a consequence of axioms which have a transparent and intuitive formulation in this particular context of health care resource allocation. These axioms address principles of fairness in 'simple' situations; once having taken an ethical position in the 'simple case', the consistent distribution method for complex situations (possibly involving individuals at many different stages of life and at many different health states) turns out as a consequence of the underlying axioms.

2 Health care resource allocation

Let $N = \{1, ..., n\}$ denote the set of individuals in the society, $n \geq 3$. Let A denote the set of conceivable *health states* for an individual, with typical elements $a_i, a_i' \in A$, where subscript i indicates that the health state is for individual i. A health state description contains typically real-valued parameters (weight, blood pressure, etc.) and we assume accordingly that there are uncountable many conceivable health states. Otherwise, we impose no other assumptions on the factors characterizing A and its mathematical structure. However, the health state description is myopic in the sense that 'prospective life-time' should not enter. Instead, this factor is explicitly given by a variable $t_i \in R_+$ is the set of conceivable life-years. The *health of individual* i is a pair $(a_i, t_i) \in A \times R_+$ containing health state a_i and prospective life-time t_i. Let $\succsim_i$ be a preference relation (complete and transitive) on $A \times R_+$ representing individual i's preferences for own health.

A QALY measure is generally a function $q_i : A \times R_+ \to R$ of the form

$$q_i(a_i, t_i) = q_i(h_i(a_i), t_i), \tag{1}$$

where $h_i : A \to R$ is a *health index*, and t_i is the number life-years spent at health state a_i. Often, it is assumed that there exists $g_i : R \to R$ such that

$$q_i(a_i, t_i) = h_i(a_i)g(t_i), \tag{2}$$

with for example $g(t_i) = \frac{1}{c} - e^{-ct_i}$, $c > 0$, or $g(t_i) = t_i$ yielding the simplest functional form of individual QALYs

$$q_i(a_i, t_i) = h_i(a_i)t_i. \tag{3}$$

The latter model is assumed by many authors including Dolan (1998, p. 40).

Write $t = (t_1, ..., t_n)$, $a = (a_1, ..., a_n)$, and $q(a, t) = (q_1(a_1, t_1), ..., q_n(a_n, t_n))$. Let $(a, t) = ((a_1, t_1), ..., (a_n, t_n))$ be a *health distribution.* $(A \times R_+)^N$ is the set of conceivable distributions of health states and life-time in the society with typical elements $(a, t), (a', t') \in (A \times R_+)^N$. A *social welfare ordering* is given by a complete and transitive relation $\succsim$ on the set of conceivable distributions of health states and life-years $(A \times R_+)^N$. For any given budget, there is a set of *feasible* distributions $F \subseteq (A \times R_+)^N$, for which the most preferred distribution(s) should be identified by the health planning authority. This is *the health care resource allocation problem.* Using the social welfare ordering $\succsim$ we obtain the most preferred distribution(s) for any given set of feasible distributions (if a maximal element exists). Thus $\succsim$ induces a *health care resource allocation method.* If the social welfare ordering admits a real-valued representation, let $U : (A \times R_+)^N \to R$ be a *social welfare function* representing $\succsim$. Moreover, we write $W : R^N \to R$, $W(q(a, t)) = U(a, t)$ if social welfare may be represented by a (Bergson-Samuelson) social welfare function depending on the level of individual QALYs only.

At this point, we should make the interpretation of life-time, t_i, clear. Do we want to interpret t_i as gains, possibly negative, relative to some (known or unknown) status quo? Or do we interpret t_i as an absolute number of life-years relative to an absolute zero? (interpreted as unborn or stillborn).

This distinction certainly matters; in the following we proceed with the latter interpretation in mind. However, in certain cases, both interpretations are valid (see also the remarks in Section 5).

Note that so far, we leave open what a 'QALY' essentially is. We do not specify if QALYs represent empirically observable measures of life quality, cardinal or ordinal utilities, or something quite different. In particular, we do not initially assume that individual utility can be written in the form (1), (2), or (3). Rather, we want to derive a convenient representation from an underlying list of empirically testable axioms.

3 Individual QALYs

QALY-based individual utility functions have been derived in several works, by introducing regularity conditions on health state lotteries, see e.g. Pliskin et al. (1980), Bleichrodt et al. (1997) and Miyamoto et al. (1998). In this section, we derive a very simple characterization of QALY-based individual preferences (3) in a situation of choice under certainty.

We will throughout make the assumption that if an individual obtains zero life-time, health state does not matter. This property has been referred to as the *zero-condition*, cf. Bleichrodt et al., Miyamoto et al..

ZERO: $(a_i, 0) \sim_i (a'_i, 0)$ for all $a_i, a'_i \in A$.

We will also make use of the following *life-time continuity* axiom for individual preferences for life-years.

CONT: Let $a_i, a'_i \in A$, $t_i, t'_i, t_i^{(k)} \in R_+$ for all $k = 1, 2, 3, \ldots$. Let $t_i^{(k)}$ be a converging sequence, $t_i = \lim_{k\to\infty} t_i^{(k)}$. If $(a_i, t_i^{(k)}) \succsim_i (a'_i, t'_i)$ for all k, then $(a_i, t_i) \succsim_i (a'_i, t'_i)$. If $(a'_i, t'_i) \succsim_i (a_i, t_i^{(k)})$ for all k, then $(a'_i, t'_i) \succsim_i (a_i, t_i)$.

Moreover, it is assumed that, for any health state, there exists a strictly positive life-time preferable to zero life-time. We will refer to this condition as the *positivity condition*.

POS: For all $a_i \in A$ there exists $t_i > 0$ such that $(a_i, t_i) \succ_i (a_i, 0)$.

Assuming positivity is not vacuous, as individuals might consider some health states worse than being dead regardless of remaining life-time. The positivity condition can be dispensed with in some cases (see Section 5).

As we will see in Theorem 1 below, a representation of individual preferences by (3) hinges crucially on an axiom of *life-time scale independence*.[3] This condition states that the ranking of a pair health descriptions (containing health states and life-years) does not change if both life-times are multiplied with the same positive number.

LSI: If $(a_i, t_i) \succsim_i (a_i', t_i')$ then $(a_i, ct_i) \succsim_i (a_i', ct_i')$ for $c > 0$.

Consider for example a person that prefers 70 life-years in a wheelchair to 60 years in perfect health. The LSI axiom asserts that the person also then prefers 35 years in a wheelchair to 30 years in perfect health an vice-versa.

Pliskin et al. derive a characterization of individual QALY-based utility using a *constant proportional trade-off* axiom similar to the LSI axiom. Here, we shall demonstrate that ZERO, CONT, and POS enables a time trade-off based foundation for QALYs as individual utilities without utilizing axioms for risk neutrality in life-years or separability of preferences for health states and life-years.

As a first step towards a characterization of QALY-based individual utility function, we will demonstrate that ZERO, POS, CONT and LSI implies the following two useful properties. *Monotonicity* (MONO) is a strong variant of POS.

MONO: Let $a_i \in A$. Then $(a_i, t_i) \succ_i (a_i, t_i')$ for all $t_i, t_i' \in R_+, t_i > t_i' \geq 0$.

Another important property is that of *comparability* (COMP). This means that there exists a health state $a_i^* \in A$ (which we will refer to as the state of 'perfect health') that can be compared to any other health state in the following sense.

COMP: Let $a_i^* \in A$. Then for all $a_i \in A$ there exists $t_i > 0$ such that $(a_i^*, t_i) \sim_i (a_i, 1)$.

We may now obtain that monotonicity and comparability follows from the preceding axioms. This observation bears similarities to a recent result by Maccheroni (2001, Proposition 5).

Lemma 1 *Let the individual preference $\succsim_i$ satisfy ZERO, POS, CONT and LSI. Then MONO and COMP are satisfied.*

[3]See Moulin (1988) for a throughout treatment of *scale independence*. Scale independence has also been referred to as *homotheticity*, see Maccheroni (2001) for a recent account.

Proof: (POS, CONT, LSI $\Rightarrow$ MONO). Assume that there exists $a_i \in A$ such that $(a_i, t_i') \succsim_i (a_i, t_i)$ for $t_i > t_i' \geq 0$. By LSI, $(a_i, \frac{t_i'}{t_i}) \succsim_i (a_i, 1)$. By LSI and transitivity, $(a_i, (\frac{t_i'}{t_i})^k) \succsim_i (a_i, 1)$ for all $k = 1, 2, 3....$ By CONT, $(a_i, 0) \succsim_i (a_i, 1)$. Thus, $(a_i, 0) \succsim_i (a_i, t_i)$ for all $t_i > 0$ by LSI, contradicting POS.

(ZERO, POS, CONT, LSI $\Rightarrow$ COMP). Let a_i^* be an arbitrary reference health state. Now assume that there is some $a_i \in A$, $a_i \neq a_i^*$ such that there exists no $t_i > 0$ for which $(a_i^*, t_i) \sim_i (a_i, 1)$. By CONT, then either i) $(a_i^*, t_i) \succ_i (a_i, 1)$ for all $t_i > 0$ or ii) $(a_i, 1) \succ_i (a_i^*, t_i)$ for all $t_i > 0$.

Case i): By CONT then $(a_i^*, 0) \succsim_i (a_i, 1)$ and by ZERO and transitivity $(a_i, 0) \succsim_i (a_i, 1)$. Thus, $(a_i, 0) \succsim_i (a_i, t_i)$ for all $t_i > 0$ by LSI, contradicting POS.

Case ii): By LSI $(a_i, \frac{1}{t_i}) \succ_i (a_i^*, 1)$ for all $t_i > 0$ then by CONT $(a_i, 0) \succsim_i (a_i^*, 1)$. By ZERO and transitivity $(a_i^*, 0) \succsim_i (a_i^*, 1)$. Thus, $(a_i^*, 0) \succsim_i (a_i^*, t_i)$ for all $t_i > 0$ by LSI contradicting POS. $\square$

Let $a_i^* \in A$ a given health state. In principle, a_i^* may be any conceivable health state, but one may prefer letting a_i^* be a state of 'perfect health'. Define the *time trade-off correspondence* $H_i : A \rightarrow 2^{R_+}$,

$$H_i(a_i) = \{t_i \mid (a_i^*, t_i) \sim_i (a_i, 1)\}. \tag{4}$$

As a corollary of Lemma 1, we observe that H_i is a single valued, non-empty function.

Corollary 1 *Let the individual preference $\succsim_i$ satisfy ZERO, CONT, POS and LSI. Then for any $a_i^* \in A$ there exists a function $h_i : A \rightarrow R_+$ satisfying*

$$h_i(a_i) = \{t_i \mid (a_i^*, t_i) \sim_i (a_i, 1)\}. \tag{5}$$

Proof: By Lemma 1, COMP is satisfied. Thus $h_i(a_i)$ is non-empty for all $a_i \in A$. As also MONO holds, $h_i(a_i)$ is a singleton. $\square$

We may now formulate the following characterization of QALY-based individual utilities.

Theorem 1 *Let the individual preference $\succsim_i$ satisfy ZERO, CONT, POS and LSI. Then there is a function $f_i : A \rightarrow R$, which is unique up to a positive linear transformation, such that*

$$(a_i, t_i) \succsim_i (a_i', t_i') \Leftrightarrow f_i(a_i)t_i \geq f_i(a_i')t_i'. \tag{6}$$

Proof: Let a_i^* be an arbitrary reference health state. By Corollary 1, h_i is a well defined TTO-function from A to R_+. Moreover, by Lemma 1 (COMP) we observe that $h_i(a_i) > 0$ for all $a_i \in A$.

As $(a_i, 1) \sim_i (a_i^*, h_i(a_i))$ by LSI, $(a_i, t_i) \sim_i (a_i^*, t_i h_i(a_i))$, and similarly $(a_i', t_i') \sim_i (a_i^*, t_i' h_i(a_i'))$. By Lemma 1 (MONO) and transitivity $(a_i, t_i) \succsim_i (a_i', t_i') \Leftrightarrow h_i(a_i)t_i \geq h_i(a_i')t_i'$. Thus (6) holds if $f_i = h_i$.

Now, assume that $f_i \neq ch_i$ for $c > 0$. Then for some pair $a_i, a_i' \in A$, $\frac{f_i(a_i)}{f_i(a_i')} \neq \frac{h_i(a_i)}{h_i(a_i')}$. Let $t_i > 0$ and let t_i' be defined such that $(a, t_i) \sim_i (a_i', t_i')$. Thus $h_i(a_i)t_i = h_i(a_i')t_i'$ but $f_i(a_i)t_i \neq f_i(a_i')t_i'$ contradicting (6). □

Conversely, if $f_i : A \to R$ is a function satisfying (6) then ZERO, CONT, and LSI hold. Note, however, that POS is not implied by the functional form of the individual utility function (see also Section 5). POS holds if and only if $f_i > 0$.

Note that individuals obeying axioms ZERO, CONT, POS, and LSI do not necessarily regard the utility of life as being proportional to prospective life-time in the sense that living for a hundred years is 'twice as good' as living fifty years at the same health state etc. We do not require assessments of preference intensity. The characterization of a QALY-based individual utility function should be considered as a tool for analytical convenience applicable under conditions weaker than requiring intensities of preferences. For example, assume that an individual regards his utility of life as represented by a utility function $f_i(a_i)\sqrt{t_i}$. As ZERO, CONT, POS, and LSI are satisfied, it follows from Theorem 1 that there exists a function $h_i(\cdot)$ such that $q_i(a_i, t_i) = h_i(a_i)t_i$ represents his preferences.

4 Aggregating QALYs

The social welfare ordering $\succsim$ satisfies the *Pareto principle*, if for any pair of distributions, if one distribution is preferred to another distribution by all individuals, then the first distribution should also be preferred according to the social ordering. Formally,

PAR: $(a, t) \succ (a', t')$ if $(a_i, t_i) \succ_i (a_i', t_i')$ for all $i \in N$.

As with individual continuity of preferences, we shall make use of a corresponding assumption of *life-time continuity of social preferences*.

CONT-S: Let $a, a' \in A^N$, $t, t^{(k)} \in R_+^N$ for all $k = 1, 2, 3....$ Let $t^{(k)}$ be a converging sequence, $t = \lim_{k \to \infty} t^{(k)}$. If $(a, t^{(k)}) \succsim (a', t')$ for all k, then $(a, t) \succsim (a', t')$. If $(a', t') \succsim (a, t^{(k)})$ for all k, then $(a', t') \succsim (a, t)$.

In the following, we shall, without loss of generality, use a common reference health state $a_i^* = a_*$ for all $i \in N$. It is then clear from Theorem 1 that if ZERO, CONT, POS and LSI holds for $i, j \in N$, then $\succsim_i = \succsim_j$ if and only if $h_i = h_j$ (where h_i and h_j are of the form (5)).

We can now formulate the basic representation theorem, which we will make repeatedly use of in the following subsections.[4]

Theorem 2 *Let individual preferences satisfy ZERO, CONT, POS and LSI for all $i \in N$. If the social welfare ordering $\succsim$ satisfies PAR and CONT-S then $\succsim$ is represented by a continuous social welfare function depending only on the distribution of individual QALYs, i.e. there exists a continuous function $W : R^N \to R$ such that*

$$(a, t) \succsim (a', t') \Leftrightarrow W(q(a, t)) \geq W(q(a', t')),$$

where $q_i(a_i, t_i) = h_i(a_i)t_i$, and h_i is the health state index (5) for all $i \in N$.

Proof: Let $q_i(a_i, t_i) = h_i(a_i)t_i$. By Theorem 1, $q_i(a_i, t_i) = q_i(a_i', t_i')$ if and only if $(a_i, t_i) \sim_i (a_i', t_i')$. Moreover, if $q_i(a_i, t_i) = q_i(a_i', t_i')$ for all $i \in N$ then $(a, t) \sim (a', t')$. To see this, assume otherwise for some $(a, t), (a', t') \in (A \times R_+)^N$ where $(a_i, t_i) \sim_i (a_i', t_i')$ that $(a, t) \succ (a', t')$. Let $t'(\varepsilon) = (t_1' + \varepsilon, ..., t_n' + \varepsilon)$. By Lemma 1 (MONO) and transitivity $(a_i', t_i' + \varepsilon) \succ_i (a_i, t_i)$ for all $\varepsilon > 0$, $i \in N$. Then by PAR, $(a', t'(\varepsilon)) \succ (a, t)$ for all $\varepsilon > 0$, but $(a, t) \succ (a', t'(0))$ contradicting CONT-S.

Let $\succsim_Q$ be the social welfare ordering on $Q = R_+^N$ induced by $\succsim$ in the sense that $q' \succsim_Q q''$ if there exists $(a', t'), (a'', t'') \in (A \times R_+)^N$, $q' = q(a', t')$ and $q'' = q(a'', t'')$, for which $(a', t') \succsim (a'', t'')$. Since $(a', t') \sim (a'', t'')$ if $q_i(a_i', t_i') = q_i(a_i'', t_i'')$ for all $i \in N$, $\succsim_Q$ is a complete and transitive social welfare ordering on Q representing $\succsim$.

It remains to show that $\succsim_Q$ is continuous. That is, for any $q' \in R^N$, and any converging sequence q^k, $\lim_{k \to \infty} q^k = q'$ we require that i) If $q^k \succsim_Q q''$ for all k then $q' \succsim_Q q''$, and ii) If $q'' \succsim_Q q^k$ for all k then $q'' \succsim_Q q'$. We will

[4]This result has also a parallel in recent work by Kaplan and Shavell (2001). Note that here it is not assumed that social welfare is represented by a social welfare function. Rather, a social welfare function is derived from underlying assumptions.

verify that case i) holds. Let (a'', t'') satisfy $q_i(a''_i, t''_i) = q''_i$ for all $i \in N$. By Lemma 1 (COMP) there exists t^k such that $q^k = \left(q_1(a_*, t^k_1), ..., q_n(a_*, t^k_n)\right)$. As q^k converges, $\lim_{k\to\infty} t^k = t'$ exists. By CONT-S, $((a_*, t'_1), ..., (a_*, t'_n)) \succsim ((a''_1, t''_1), ..., (a''_n, t''_n))$.

As case ii) follows by a similar argument, we may conclude that $\succsim_Q$ is continuous. Thus $\succsim_Q$ can be represented by a continuous social welfare function $W : R^N \to R$ (cf. Proposition 3.C.1 in Mas-Colell, Whinston and Green, 1995). □

By Theorem 2, we observe that under a set of very mild assumptions, we may restrict attention to continuous social welfare functions depending only on the level of individual QALYs.

4.1 Dolan's solution: a case of age discrimination

Up to this point, we have only introduced relatively mild assumptions for the social welfare ordering: the Pareto principle and continuity. In what follows, we will demonstrate that Dolan's solution may be derived as a result of the following axiom, which turns out to be very powerful in combination with the QALY-based representation of the individuals' preferences. The axiom says that for any health distribution, any permutation of life-times between the individuals (keeping health states fixed) may be performed without affecting social welfare. This condition will be referred to as *life-time anonymity*.

LTA: $(a, t) \sim (a, t_\sigma)$ for all $(a, t) \in (A \times R_+)^N$, and all permutations $\sigma : N \to N$.

As we for the sake of generality put no restrictions on the set of conceivable life-years (R_+), we shall make use of the following assumption denoted *full health domain* which assures that health state values between 0 and 1 may not be ruled out.

FHD: $h_i(A) \supseteq]0, 1]$.

We are now able to characterize Dolan's solution to the health care resource allocation problem.

Theorem 3 *Let individual preferences satisfy ZERO, CONT, POS, LSI, and FHD for all $i \in N$. Then PAR, CONT-S, and LTA are satisfied if* *and only if*

$$(a,t) \succsim (a',t') \Leftrightarrow \prod_{i\in N} h_i(a_i)t_i \geq \prod_{i\in N} h_i(a_i')t_i',$$

where h_i is the health state index (5) for all $i \in N$.

Proof: *Sufficiency* ($\Rightarrow$). By Theorem 2, the social welfare ordering $\succsim$ may be represented by a continuous social welfare function $W(q_1,...,q_n)$, $q_i = h_i(a_i)t_i$. By PAR $W(q_1,...,q_n) > W(q_1',...,q_n')$ if $q_i > q_i'$ for all $i \in N$; thus W is monotone. Note that by LTA, $W(h_1t_1,...,h_nt_n) = W(h_1t_{\sigma(1)},...,h_nt_{\sigma(n)})$ for all $h,t \in R_+^N$.

Consider an arbitrary distribution $(q_1,...,q_n)$. Let p be the number of unordered pairs $i,j \in N$, $i \neq j$ (so that $p = 1+2+....+(n-1)$). Let $P = \{1,...,p\}$ be a list of the unordered pairs where each pair appears exactly once. Let k denote "pair number k", $k \in \{1,...,p\}$.

Now we define $q(k)$ recursively as follows:

$q(0) \equiv (q_1,...,q_n)$,

$q(1)$ is the distribution obtained after the following redistribution. Let (i,j) be pair number 1. Then

$$q(1) = \left(q_1,...,(\sqrt[n]{q_i})^{n-1}(\sqrt[n]{q_j}),...,(\sqrt[n]{q_j})^{n-1}(\sqrt[n]{q_i}),...,q_n\right).$$

Let (i',j') be pair number $k+1$. Given $q(k)$ then define $q(k+1)$ as

$$q(k) = \left(q_1(k),...,\frac{q_{i'}(k)}{\sqrt[n]{q_{i'}}}(\sqrt[n]{q_{j'}}),...,\frac{q_{j'}(k)}{\sqrt[n]{q_{j'}}}(\sqrt[n]{q_{i'}}),...,q_n(k)\right).$$

Note that

$$q_i(p) = \frac{q_i}{(\sqrt[n]{q_i})^{n-1}}(\sqrt[n]{q_1})\cdots(\sqrt[n]{q_{i-1}})(\sqrt[n]{q_{i+1}})\cdots(\sqrt[n]{q_n}),$$

thus

$$q_i(p) = \sqrt[n]{q_1}\sqrt[n]{q_2}\cdots\sqrt[n]{q_n},$$

for all $i \in N$. We claim that $q(k) \sim q(k+1)$ for all k. For this, let again (i',j') denote pair number $k+1$, and for $c > 0$ define

$$\overline{h}_{i'} = \frac{q_{i'}(k)}{c\sqrt[n]{q_{i'}(k)}},\ t_{i'} = c\sqrt[n]{q_{i'}(k)},$$

and

$$\overline{h}_{j'} = \frac{q_{j'}(k)}{c\sqrt[n]{q_{j'}(k)}},\ t_{j'} = c\sqrt[n]{q_{j'}(k)}.$$

For c sufficiently large $\overline{h}_{i'}, \overline{h}_{j'} \in \left]0,1\right]$, and by FHD there exist health states $a_{i'}$,$a_{j'}$ such that $h_{i'}(a_{i'}) = \overline{h}_{i'}$ and $h_{j'}(a_{j'}) = \overline{h}_{j'}$. Hence by LTA $q(k) \sim q(k+1)$. From transitivity we conclude that $q \sim q(p)$.

Since

$$\begin{aligned} W(q_1,...,q_n) &= W(\sqrt[n]{q_1}\sqrt[n]{q_2}\cdots\sqrt[n]{q_n},...,\sqrt[n]{q_1}\sqrt[n]{q_2}\cdots\sqrt[n]{q_n}) \\ &= \overline{W}(\sqrt[n]{q_1}\sqrt[n]{q_2}\cdots\sqrt[n]{q_n}), \end{aligned}$$

we have by monotonicity that $W(q) = F(q_1 q_2 \cdots q_n)$, where F is some strictly increasing transformation.

Necessity ($\Leftarrow$). It is easily verified that the axioms are implied by the social welfare function $U(a,t) = \prod_{i\in N} h_i(a_i)t_i$. □

Consider for example two individuals at the same age each facing prospective life-time of 20 years. Assume that individual 1 is at a health state a_1 with no severe disabilities or pains, and individual 2 suffers hard at health state a_2 being considered only slightly better than death. Assume that individual 1 attributes the health state value $h_1(a_1) = 1$ to health state a_1, whereas individual 2 judges his own health state value as being $h_2(a_2) = 0.1$. Now, assume that due to a new, capacity constrained, technology we are able to prolong the life-time with, say, 10 years for one of the individuals (remaining at unchanged health state). According to Dolan's solution, it is socially equally desirable to provide health care to obtain extra life-years to either individual. The allocation of life-years does not depend on the individuals' actual health states.

Consider the following axiom of *relative life-time comparisons.*

RLC: There exists $a_- \in A$ such that for all $i,j \in N, a_i = a_j = a_-, c > 0$

$$\left[(a_-,ct_i),(a_-,t_j),(a_k,t_k)_{k\in N\setminus\{i,j\}}\right] \sim \left[(a_-,t_i),(a_-,ct_j),(a_k,t_k)_{k\in N\setminus\{i,j\}}\right].$$

Axiom RLC asserts that for two individuals at the same health state it is socially equally desirable to extend the life of either individual with a number of life-years *proportional* to their current prospective life-years.

Under our previous assumptions RLC may alternatively replace LTA in Theorem 3.[5]

Theorem 4 *In Theorem 3 we may replace FHD and LTA with RLC.*

Proof: Let individual preferences satisfy ZERO, CONT, POS, LSI for all $i \in N$; moreover assume that the social welfare ordering satisfies PAR and CONT-S. We claim that RLC is satisfied if and only if the social welfare function is of the form $U(a,t) = \prod_{i \in N} h_i(a_i)t_i$.
Sufficiency ($\Rightarrow$). Since

$$\left[(a_-, ct_i), (a_-, t_j), (a_k, t_k)_{k \in N \setminus \{i,j\}}\right] \sim \left[(a_-, t_i), (a_-, ct_j), (a_k, t_k)_{k \in N \setminus \{i,j\}}\right],$$

for all $t_i, t_j \geq 0,$ we have by Theorem 1 and 2 that

$$W\left(ch_i(a_-)t_i, h_j(a_-)t_j, ..., h_k(a_k)t_k\right) = W\left(h_i(a_-)t_i, ch_j(a_-)t_j, ..., h_k(a_k)t_k\right),$$

where W represents the social welfare ordering. That is, for any vector $(q_1, ..., q_n) \geq 0$,

$$W\left(cq_i, q_j, q_{k \in N \setminus \{i,j\}}\right) = W\left(q_i, cq_j, q_{k \in N \setminus \{i,j\}}\right);$$

which in turn implies

$$W\left(q_1, ..., q_n\right) = W\left(q_1 q_2 ... q_n, 1, ..., 1\right).$$

Since W is monotone we find that

$$W = F\left(\prod_{k \in N} q_k\right),$$

for some strictly increasing transformation F.
Necessity ($\Leftarrow$). It is easily verified that the axioms are implied by the social welfare function $U(a,t) = \prod_{i \in N} h_i(a_i)t_i$. □

As demonstrated, Dolan's solution has a foundation in distributive justice ethics. A difficulty here is that with a multiplicative social welfare function, persons with very low life-times (if not treated) may be given unreasonable

[5] We may therefore observe, that the RLC condition is sufficient to establish a scale independent social welfare function (d'Aspremont and Gevers, 1977; Moulin, 1988).

high priority. As the prospective life-time (before treatment) gets closer to zero, this bias becomes absurd. This problem does, however, not occur if one restricts attention to some minimum level of life-time. Nevertheless, many people would probably find that *ceteris paribus* it is preferable to prolong the life of an individual at a bearable health state instead of prolonging the life of an individual at a painful or depressed health state.

4.2 Utilitarianism: the case of non-age discrimination

Dolan's solution admits a very strong aversion to inequality in life-times within a population. For example, using Dolan's solution it becomes socially more desirable to prolong the life of a one year old child with one extra year than extending 10 fifteen years old persons with one year each. This prioritization carries through regardless of the specific health states of the persons. Dolan's solution can therefore be considered a variant of the 'fair innings' reasoning (see Harris, 1985; Williams, 1997), reflecting the viewpoint that health planning authorities should attribute a suitable number of (quality adjusted) life-years to each person, the average number of (quality adjusted) life-years, and little effort should be carried out by the health planning authority to improve or extend the life of persons at age beyond the average. The ethics suggested by Dolan's solution may accordingly be viewed as being too harsh. Discrimination against the elderly may seem difficult to justify if the prospective life-years are the same, and one might find a non-discriminative point of view more desirable.

We formulate *non-age discrimination* as following.

NAD: There exists $a_- \in A$ such that for all $i, j \in N, a_i = a_j = a_-, c > 0$

$$\left[(a_-, t_i + c), (a_-, t_j), (a_k, t_k)_{k \in N \setminus \{i,j\}}\right] \sim \left[(a_-, t_i), (a_-, t_j + c), (a_k, t_k)_{k \in N \setminus \{i,j\}}\right].$$

It may appear reasonable to permit age-discrimination on the ground that the number of remaining life-years (and thereby the number of years the treatment can be enjoyed) is less for an old individual than for a young individual facing the same treatment for similar complaints. However, one may hold the view that an individual is not less worthy of treatment on the *sole* ground that the individual has already lived for sufficiently many years. The NAD axioms states that this view holds true for at least one particular health state. With this version of non-discrimination we have the following characterization.

Theorem 5 *Let individual preferences satisfy ZERO, CONT, POS and LSI for all $i \in N$, and let the social welfare ordering satisfy PAR and CONT-S. Then NAD is satisfied if and only if*

$$(a,t) \succsim (a',t') \Leftrightarrow \sum_{i \in N} h_i(a_-)h_i(a_i)t_i \geq \sum_{i \in N} h_i(a_-)h_i(a_i')t_i',$$

where h_i is the health state index (5) for all $i \in N$.

Proof: *Sufficiency* ($\Rightarrow$). By Theorem 1 and 2, the social welfare ordering may be represented by a continuous function $W(q_1,...,q_n)$, $q_i = h_i(a_i)t_i$, where h_i satisfies (5) with $a_i^* = a_*$. By PAR, $W(q_1,...,q_n) > W(q_1',...,q_n')$ if $q_i > q_i'$ for all $i \in N$, i.e. W is monotonic.

We shall demonstrate that $W(q_1,...,q_n) = W(q_1',...,q_n')$ if and only if $\sum_{i \in N} q_i = \sum_{i \in N} q_i'$. By Theorem 1 and 2

$$\left[(a_-, t_i + c), (a_-, t_j), (a_k, t_k)_{k \in N \setminus \{i,j\}}\right] \sim \left[(a_-, t_i), (a_-, t_j + c), (a_k, t_k)_{k \in N \setminus \{i,j\}}\right]$$

for all $i,j \in N$ if and only if

$$\left[(a_*, h_i(a_-)(t_i + c)), (a_*, h_j(a_-)t_j), (a_*, h_k(a_k)t_k)_{k \in N \setminus \{i,j\}}\right] \sim$$

$$\left[(a_*, h_i(a_-)t_i), (a_*, h_j(a_-)(t_j + c)), (a_*, h_k(a_k)t_k)_{k \in N \setminus \{i,j\}}\right]$$

for all $i,j \in N$. Now, let $\overline{c}_k = h_k(a_-)c$, and $\overline{t}_k = h_k(a_k)t_k$ for all $k \in N$ and we obtain equivalently

$$\left[(a_-, t_i + c), (a_-, t_j), (a_k, t_k)_{k \in N \setminus \{i,j\}}\right] \sim \left[(a_-, t_i), (a_-, t_j + c), (a_k, t_k)_{k \in N \setminus \{i,j\}}\right]$$

for all $i,j \in N$ if and only if

$$\left[(a_*, \overline{t}_i + \overline{c}_i), (a_*, \overline{t}_j), (a_*, \overline{t}_k)_{k \in N \setminus \{i,j\}}\right] \sim \left[(a_*, \overline{t}_i), (a_*, \overline{t}_j + \overline{c}_j)), (a_*, \overline{t}_k)_{k \in N \setminus \{i,j\}}\right]$$

for all $i,j \in N$. By Theorem 2, we may thus observe that the social welfare function satisfies $W(q_i + \overline{c}h_i(a_-), q_j, q_{k \in N \setminus \{i,j\}}) = W(q_i, q_j + \overline{c}h_j(a_-), q_{k \in N \setminus \{i,j\}})$ for all $i,j \in N$, implying that $W(q_i, q_j, q_{k \in N \setminus \{i,j\}}) = W(q_i + \overline{c}, q_j - \overline{c}, q_{k \in N \setminus \{i,j\}})$ for all $i,j \in N$, $q_j - \overline{c} \geq 0$. Repeating this argument, we obtain

$$W(q_i, q_j, q_{k \in N \setminus \{i,j\}}) = W(\sum_{i \in N} h_i(a_-)q_i, 0, ..., 0).$$

Thus by monotonicity $W(q_1, ..., q_n) = W(q'_1, ..., q'_n)$ if and only if $\sum_{i \in N} h_i(a_-)q_i = \sum_{i \in N} h_i(a_-)q'_i$.
Necessity ($\Leftarrow$). It is easily verified that NAD is implied by the social welfare function $U(a,t) = \sum_{i \in N} h_i(a_-)h_i(a_i)t_i$. □

As an important special case we observe that if NAD holds for the reference health state a_* (i.e. $a_* = a_-$) then $h_i(a_-) = 1$ for all i, and we obtain unweighted QALY-utilitarianism, $U(a,t) = \sum_{i \in N} h_i(a_i)t_i$.

4.3 Other aggregation methods

For sake of simplicity, it will in this section be assumed that individual QALYs are always strictly positive. If neither axiom LTA nor axiom NAD is acceptable, the health planning authority may want to look for alternative solutions to the health care resource allocation problem. Which requirements should be fulfilled by an acceptable social welfare ordering? A well known principle is that of *separability*.

SEP: $\left[(a_i, t_i)_{i \in S}, (a_i, t_i)_{i \in N \backslash S}\right] \succsim \left[(a'_i, t'_i)_{i \in S}, (a_i, t_i)_{i \in N \backslash S}\right] \Leftrightarrow$
$\left[(a_i, t_i)_{i \in S}, (a'_i, t'_i)_{i \in N \backslash S}\right] \succsim \left[(a'_i, t'_i)_{i \in S}, (a'_i, t'_i)_{i \in N \backslash S}\right]$, for all $S \subseteq N$.

A social welfare ordering is separable if the social preferences for distributions of health (health states and life-years) within a subgroup S of N, does not depend on the health of the complement group $N \backslash S$. Due to Theorem 2, we may restrict attention to continuous social welfare functions depending on individual QALYs only. We can therefore immediately formulate the following variant of a theorem by Debreu (1960). Theorem 6 states that social preferences may be represented by a real-valued function being additively separable in the level of individual QALYs.

Theorem 6 *Let individual preferences satisfy ZERO, CONT, POS and LSI for all $i \in N$, and let the social welfare ordering satisfy PAR, CONT-S and SEP. Then there exist functions $v : R \to R$, and a social welfare function $W : R^N \to R$,*

$$W(q_1, ..., q_n) = \sum_{i \in N} v(q_i),$$

satisfying

$$(a,t) \succsim (a',t') \Leftrightarrow \sum_{i \in N} v(h_i(a_i)t_i) \geq \sum_{i \in N} v(h_i(a'_i)t'_i),$$

where h_i is the health state index (5) for all $i \in N$.

Recall the LSI condition for individual preferences for own health: this condition states that the ranking of a pair of health descriptions does not change if both life-times are multiplied with the same positive constant. The condition translates directly to the social welfare ordering. A social welfare ordering satisfies *common life-time scale independence* if the ranking of a pair of health distributions does not change when all life-times are multiplied with the same positive constant. Formally,

CLSI: If $(a,t) \succsim (a',t')$ then $(a,ct) \succsim (a',ct')$ for $c > 0$.

If it is assumed that the individuals' preferences satisfy LSI, it seems natural that the life-time scale independence extends to the social welfare ordering. As an application of a result due to Roberts (1980),[6] we obtain that the preceding axioms impose quite narrow restrictions on the functional form of the social welfare function.

Theorem 7 *Let individual preferences satisfy ZERO, CONT, POS and LSI for all $i \in N$, and let the social welfare ordering satisfy PAR and CONT-S. Then SEP and CLSI are satisfied if and only if the social welfare ordering is represented (up to a monotone transformation) by one of the following social welfare functions.*

1. $U(a,t) = \sum_{i \in N} \alpha_i \left(h_i(a_i)t_i\right)^p$, $p > 0$,
2. $U(a,t) = -\sum_{i \in N} \alpha_i \left(h_i(a_i)t_i\right)^p$, $p < 0$,
3. $U(a,t) = \sum_{i \in N} \alpha_i \ln\left(h_i(a_i)t_i\right)$,

where h_i is the health state index (5) for all $i \in N$.

Proof: *Sufficiency* ($\Rightarrow$). By Theorem 1, for any individual i there exists $h_i : A \rightarrow R_+$ such that $q_i = h_i(a_i)t_i$ represents individual preferences for own health. By Theorem 2, there exists a continuous welfare function $W = W(q)$ representing the social welfare ordering. By Theorem 6, W is additive. As $h_i(a_i)ct_i = cq_i$, CLSI translates to $W(q_1,...,q_n) \geq W(q_1',...,q_n') \Leftrightarrow W(cq_1,...,cq_n) \geq W(cq_1',...,cq_n')$. By Roberts (1980) a continuous, additive function W satisfying this property has one of the following functional forms

[6] See also Moulin (1988).

1. $W(q_1, ..., q_n) = \sum_{i \in N} \alpha_i q_i^p$, $p > 0$,

2. $W(q_1, ..., q_n) = -\sum_{i \in N} \alpha_i q_i^p$, $p < 0$,

3. $W(q_1, ..., q_n) = \sum_{i \in N} \alpha_i \ln q_i$.

Necessity ($\Leftarrow$). It is easily verified that the axioms are implied by each social welfare function. □

This class of functions has also been studied by Atkinson (1970) as measures of inequality (see also Dolan, 1998, p. 44). Here, we have shown that by extending the life-time scale independence condition characterizing individuals' preferences to a common life-time scale independence condition for social welfare orderings, one obtains a persuasive derivation of this particular class of social welfare functions given that the basic QALY representation of individual preferences (3) holds true. Of course, stronger axioms are required for narrowing down this class of social welfare orderings to a unique social welfare ordering. LTA and NAD are examples of two such axioms.

For practical purposes and empirical tests, a major simplification is obtained if comparisons between individuals, in terms of trade-offs for life-times, are required only for individuals at identical health states. As we put no restrictions on the complexity of the health state factors, comparing individuals at different health states may become very difficult. On the contrary, life-times are unidimensional measures relatively simple to compare. The following axiom denoted *common health state independence* states that if the health states of a pair of individuals are identical, trade-offs comparisons for their life-times does not depend on the particular health state.

CHI: If $a_i = a_j = a_- \in A$ and $\left[(a_-, t_i), (a_-, t_j), (a_k, t_k)_{k \in N \setminus \{i,j\}}\right] \succsim \left[(a_-, t_i'), (a_-, t_j'), (a_k, t_k)_{k \in N \setminus \{i,j\}}\right]$ then $\left[(a_-', t_i), (a_-', t_j), (a_k, t_k)_{k \in N \setminus \{i,j\}}\right] \succsim \left[(a_-', t_i'), (a_-', t_j'), (a_k, t_k)_{k \in N \setminus \{i,j\}}\right]$ for all $a_-' \in A$.

If axiom FHD is included in the list of basic properties characterizing individual preferences for own health, we may replace CLSI with CHI. We shall, however, also make use of a *symmetry* axiom stating that the individuals have identical preferences.

SYM: $(a_i, t_i) \succsim_i (a_i', t_i') \Leftrightarrow (a_i, t_i) \succsim_j (a_i', t_i')$ for all $i, j \in N, (a_i, t_i) \in A \times R_+$.

When the symmetry axiom holds, let $h = h_i$ be the common health state index (with $a_* = a_i^*$ as the common reference health state.)

Theorem 8 *If SYM and FHD holds, then in Theorem 7 we may replace CLSI by CHI with* $\alpha_1 = ... = \alpha_n = 1$.

Proof: Let individual preferences satisfy ZERO, CONT, POS, LSI, and FHD for all $i \in N$, and let the social welfare ordering satisfy PAR and CONT-S and SEP.

(CLSI⇒CHI). Assume that CLSI holds. By Theorem 6 we have that if

$$\sum_{i \in N} v(h(a_i)t_i) \geq \sum_{i \in N} v(h(a_i')t_i')$$

then

$$\sum_{i \in N} v(h(a_i)ct_i) \geq \sum_{i \in N} v(h(a_i')ct_i'),$$

for all $c > 0$. Thus, if

$$v(h(a_i)t_i) + v(h(a_j)t_j) \geq v(h(a_i)t_i') + v(h(a_j)t_j')$$

then

$$v(h(a_i)ct_i) + v(h(a_j)ct_j) \geq v(h(a_i)ct_i') + v(h(a_j)ct_j').$$

Therefore, if $a_i = a_j = a_-$ and

$$v(h(a_-)t_i) + v(h(a_-)t_j) \geq v(h(a_-)t_i') + v(h(a_-)t_j')$$

then

$$v(h(a_-')t_i) + v(h(a_-')t_j) \geq v(h(a_-')t_i') + v(h(a_-')t_j'),$$

for all $a_-' \in A$, since by MONO we may choose $c > 0$ such that $h(a_-') = h(a_-)c$. Thus CHI holds.

(CHI⇒CLSI). By Theorem 6 there are real-valued functions $v_1, ..., v_n$ such that $W(q_1, ..., q_n) = \sum_{i \in N} v(q_i)$ represents the social welfare ordering. Now, consider $q_1, ..., q_n$ and $q_1', ..., q_n'$ and $c > 0$. By SYM and FHD we may choose health states a and a_- such that $0 < h(a_-), h(a_-') \leq 1$ and $h(a_-) = ch(a_-')$. Now, let $t_1, ..., t_n$ and $t_1', ..., t_n'$ be such that $h(a_-)t_i = q_i$ and $h(a_-)t_i' = q_i'$ for all i. By CHI we have that

$$\sum_{i \in N} v(h(a_-)t_i) \geq \sum_{i \in N} v(h(a_-)t_i')$$

implies

$$\sum_{i \in N} v(h(a'_-)t_i) \geq \sum_{i \in N} v(h(a'_-)t'_i),$$

or equivalently:

$$\sum_{i \in N} v(q_i) \geq \sum_{i \in N} v(q'_i)$$

implies that

$$\sum_{i \in N} v(cq_i) \geq \sum_{i \in N} v(cq'_i),$$

thus CLSI holds. □

5 Concluding remarks

Naturally, individuals' prospective health states and life-years may change (unexpectedly) over time. For the purpose of measuring a population's overall health over time (see Torrance, 1976), extensions of the present model would be relevant. Using a utilitarian social welfare function, we may overcome this difficulty by considering *gains* and *losses* in health of the population, instead of considering health distributions compared to the natural zero of no life-years to no individuals. This might turn out to be a conceptual advantage, since the status quo health of a population could very well be unknown. We are then able to deal with health states worse than death (and thereby may avoid the positivity axiom POS), since we then obtain a natural interpretation of negative values for the TTO-functions $h_i(\cdot)$. It remains an open question how to modify Dolan's method to deal in a reasonable way with health states that are worse than being dead.

The results outlined in the previous sections provide a basis for the following approach to the health care resource allocation problem: Using TTO questionnaires, the health planning authority question representative individuals about preferences for *own* health. Moreover, the individuals are questioned to evaluate the plausibility of axioms for health care allocation: LTA, NAD, SEP, CHI etc. By using this type of questionnaires, the health planning authority might be able to avoid asking direct questions regarding the individuals' preferred shape of the social welfare function. For untrained representative individuals, it might be difficult conceptually to distinguish between the individual utility function and the social welfare function for health care resource allocation, and the combined effect of the curvatures of these functions. Using an axiomatic approach, we may obtain one way of getting around these difficulties.

To sum up, QALY-based approaches where a social welfare function depends on individual QALYs only *do* have welfare theoretic foundations. The public opinion on the plausibility of the axioms for distributive justice remains an open question for empirical investigation.

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